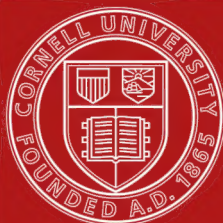


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The Mannheim & Polyphase Slide Rules

A Self Teaching Manual
with
tables of settings, equivalents and gauge points

By
William E. Breckenridge, A. M.
Associate in Mathematics
Columbia University
New York City

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K & E MANNHEIM SLIDE RULE.

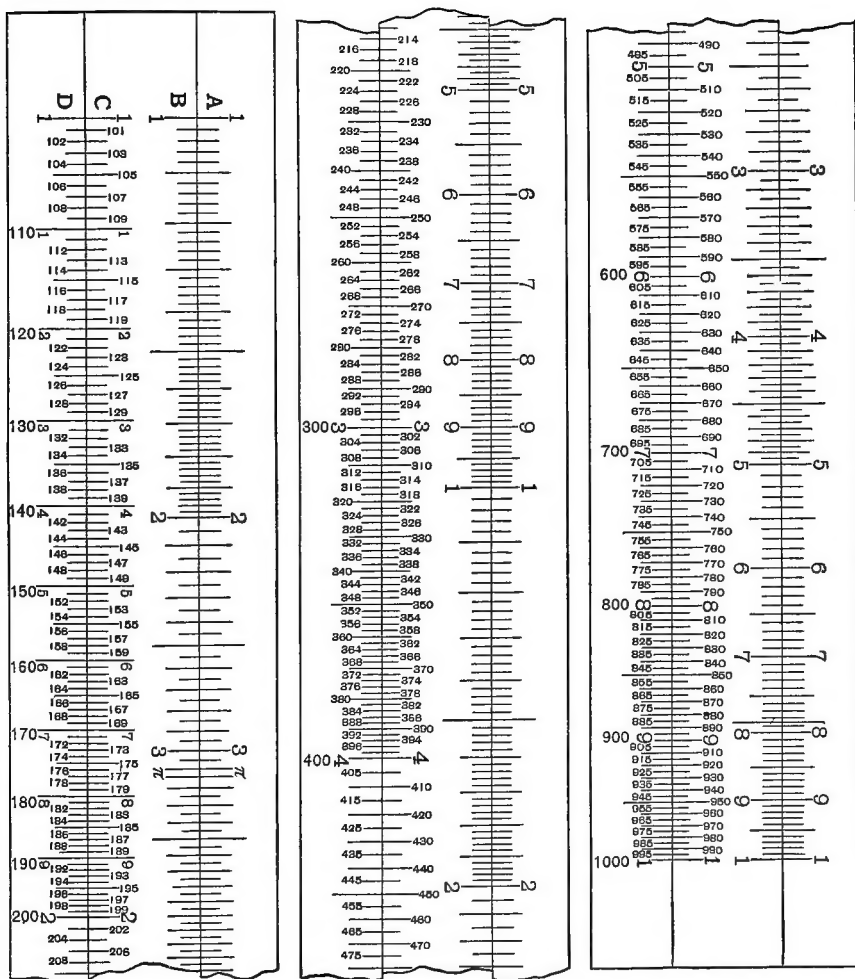
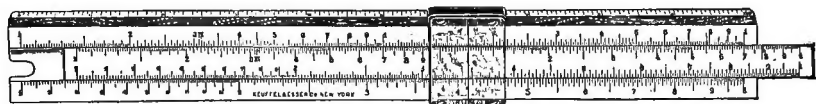
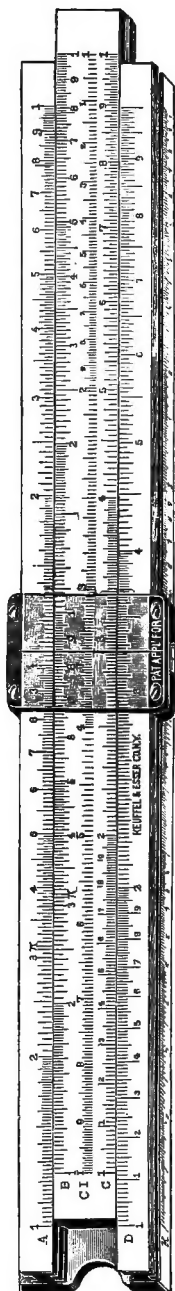


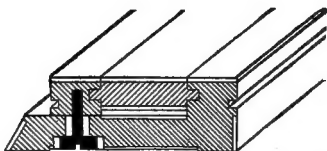
Diagram illustrating the reading of the graduations of the rule.



The Slide Rule in its present form has become an indispensable aid not only to the engineer and scientist, but also to the manufacturer, the merchant, accountant and all others whose occupation or business involves calculations.

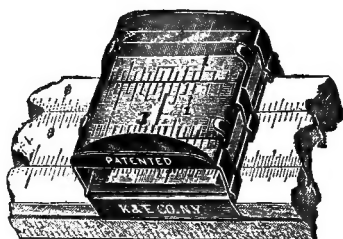
We manufacture slide rules and devote to them a separate department of our factory which is thoroughly equipped with the most improved special machinery.

Several of our improvements are protected by patents, and are, therefore, not embodied in other Rules.



Cross section of K & E Mannheim Slide Rule showing slide adjustment.

← Polyphase Slide Rule
No. 4058-3.



Magnifier for
K & E Mannheim Slide Rule.



KEUFFEL & ESSER CO., NEW YORK

The Mannheim & Polyphase Slide Rules

COMPLETE MANUAL
with
tables of settings, equivalents and gauge points

By
William E. Breckenridge, A. M.
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THE SLIDE RULE.

PREFACE.

This manual is designed to meet the needs of all who desire to learn the use of the slide rule.

Chapter I. is self-teaching by means of numerous cuts and examples simply explained. For some persons a few lessons in this chapter will teach all that is desired.

It is suggested that everyone learning to use the slide rule begin by working the examples in Chapter I.

In Chapter II., a simple explanation of the theory of the slide rule is followed by the advanced subjects of Cubes, Cube Root, Sines, Cosines, Tangents and Logarithms.

Special work for technical men and typical problems from various occupations are presented in Chapter III.

Who should use the Slide Rule?

- I. Teachers in the following types of schools:
 1. Elementary Schools as early as the eighth grade.
 2. Junior High Schools for part of their practical mathematics.
 3. High Schools in connection with logarithms, practical mathematics, or trigonometry.
 4. Colleges in their courses in algebra or trigonometry. Many colleges have already made the slide rule a part of the trigonometry course.
 5. Evening schools will hold the students better while teaching the slide rule than any other subject.
 6. Engineering and Trade Schools find the rule indispensable.
- II. Engineers, Mechanics, Chemists, and Architects have long understood its value.
- III. Private Secretaries can check reports by the slide rule in a small fraction of the time required by ordinary calculation.
- IV. Estimators, Accountants and Surveyors can make approximate calculations rapidly and with sufficient accuracy to check gross errors.

By means of the slide rule, all manner of problems involving multiplication, division and proportion can be correctly solved without mental strain and in a small fraction of the time required to work them out by the usual "figuring."

For instance, rapid calculation is made possible in such problems as the following, which are of every-day occurrence in office and shop: Estimating, discounts, simple and compound interest, converting feet into meters, pounds into kilograms, foreign money into U. S. money, the taking of a series of discounts from list prices, adding profits to costs; while dozens of equivalents are instantly shown, such as cubic inches or feet in gallons, and vice versa; centimeters in inches, inches in yards or feet; kilometers in miles, square centimeters in square inches, liters in cubic feet, kilograms in pounds; pounds in gallons; feet per second in miles per hour; circumferences and diameters of circles.

How much education is necessary?

Anyone who has a knowledge of decimal fractions can learn to use the slide rule.

How much time will it take?

The simplest operations may be learned in a few minutes, but it is recommended that at least the examples in Chapter I. be worked thoroughly and checked by the answers, in order to gain accuracy and speed. This will take from one to ten hours, according to the previous training of the student.

How accurate is the Slide Rule?

With the ten-inch slide rule results can be obtained correct to three and sometimes four significant figures.

With the twenty-inch rule, results are correct to four figures.

How to use this manual.

For the man who desires to perform the simplest operations of multiplication and division, the first few lessons in Chapter I will be sufficient. Work the illustrative examples and as many others for practice as seem necessary to obtain accuracy and speed.

For educational use, Chapter II. furnishes the necessary theory and history of the rule, while Chapter I. provides additional examples for practice. Chapter III. may be used for advanced work.

It is recommended that the users of the Polyphase Slide Rule carefully read page 73, etc. before beginning the general study of the slide rule outlined in this book.

CHAPTER I.

ESSENTIALS SIMPLY EXPLAINED.

The slide rule is an instrument that may be used for saving time and labor in most of the calculations that occur in the practical problems of the business man, mechanic, draftsman, engineer, or estimator.

There are four scales on the rule, *A*, *B*, *C*, and *D*, as shown in Fig. 2. Scales *A* and *B* are graduated from 1 to 100, the left half of the rule extending from 1 to 10. Scales *C* and *D* are graduated from 1 to 10.

In order that you may see how the rule is used on simple problems where you know the answers, let us take the following:

1. 2×3 .

Set 1 on scale *C* opposite 2 on scale *D*. Then move the glass runner to 3 on scale *C*. Directly below this 3 you will find 6, the answer.

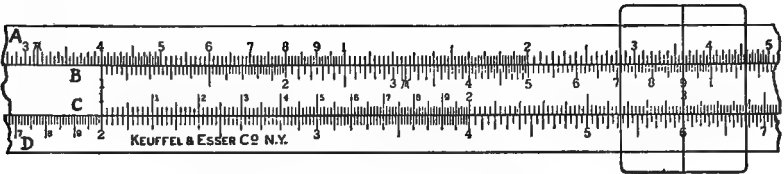


Fig. 2. $2 \times 3 = 6$, or $6 \div 3 = 2$.

2. 2×2 .

3. 2×4 . (See Fig. 3).

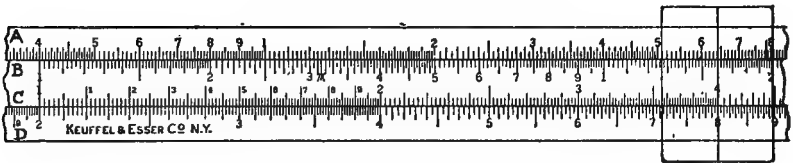


Fig. 3. $2 \times 4 = 8$ or $8 \div 4 = 2$.

4. 3×3 . (See Fig. 4).

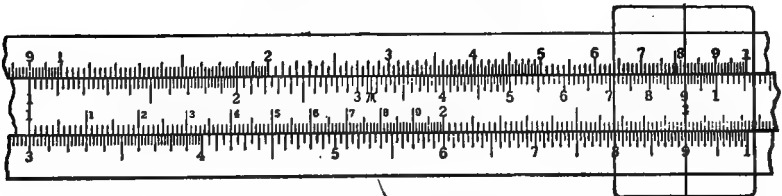


Fig. 4. $3 \times 3 = 9$, or $9 \div 3 = 3$.

5. $6 \div 3$. (See Fig. 2).

Opposite 6 on scale *D*, set 3 on scale *C*. Look along *C* to the left, till you come to 1 at the end of the slide. Under this 1 you will find 2, the answer, on scale *D*.

6. $8 \div 4$. (See Fig. 3).

7. $9 \div 3$. (See Fig. 4).

It will be noted that the cuts shown are not in the same scale. This arrangement is for the purpose of illustrating various lengths of the rule.

SQUARES AND SQUARE ROOTS

You will remember that to square a number means to multiply that number by itself; *e. g.*, 3^2 means $3 \times 3 = 9$.

8. On the slide rule this would be done as follows: Set the hair line of the glass runner to 3 on scale *D*. Above, on scale *A*, opposite the hair line, you will find 9. (Fig. 5).

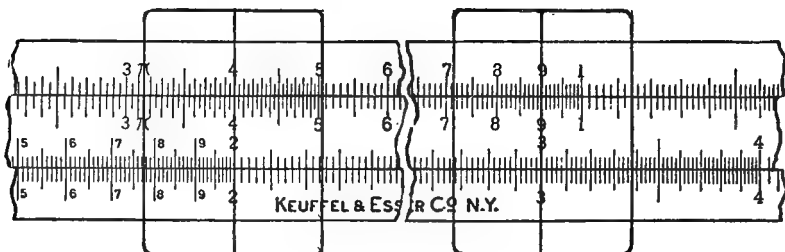


Fig. 5. $3^2 = 9$ and $2^2 = 4$.

9. In the same way find 2^2 . (See Fig. 5).

To find square roots simply do the work in the reverse order.

10. Find the square root of 9; *i. e.*, find the number which multiplied by itself will give 9.

The square root of 9 is indicated thus: $\sqrt{9}$.

Set the hair line of the runner to 9 on scale *A*, being careful to use the 9 on the left-hand half of the rule, because the other 9 is really 90. Below, on scale *D*, find 3, the answer. (Fig. 5).

11. $\sqrt{4}$.

Set the runner to 4 on *A*. Opposite the hair line on scale *D*, find 2, the answer. (Fig. 5).

We shall now proceed to apply the same methods to numbers of two or more figures.

12. 2×1.5 .

Set 1 on *C* to 2 on *D*. Move the runner to 1.5 on *C*. This will be half way between 1 and 2. Under the hair line of the runner, find 3 on *D*.

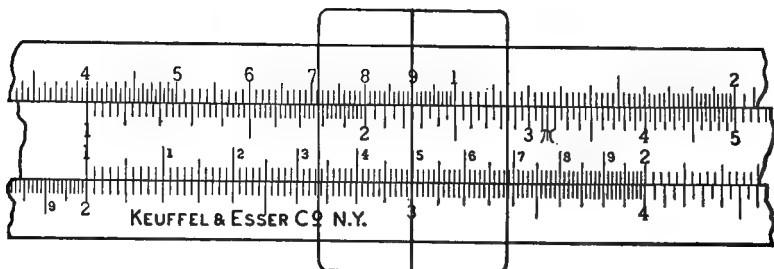


Fig. 6. $2 \times 1.5 = 3$, and $2 \times 1.8 = 3.6$.

13. 2×1.8 . Using Fig. 6, see if you can make it 3.6. The tenths on *D* are the large subdivisions.

14. 1.5×2.5 . Set 1 on *C* to 1.5 on *D*. Move the runner to 2.5 on *C*. Below 2.5, find 3.75, the answer, on *D*. Note that this answer is half way between 3.7 and 3.8, which makes it 3.75. (Fig. 7).

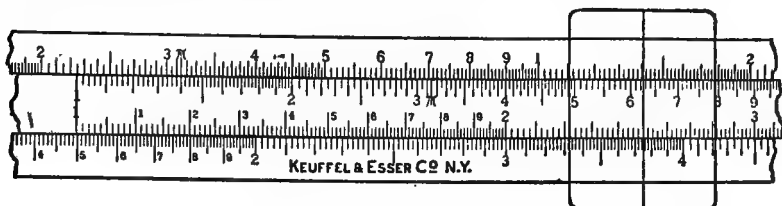


Fig. 7. $1.5 \times 2.5 = 3.75$.

15. 2×15 .

This is worked on the rule exactly like example 12, but you can see by looking at the problem that the answer is 30 and not 3.

All of these problems are worked like Example 12. As far as the slide rule is concerned, we multiply 2 by 1.5 and get 3. Then we place the decimal point by inspection. From arithmetic we remember that in multiplying decimals we first multiply as though there were no decimal point, then point off as many decimal places in the answer as there are decimal places in both numbers to be multiplied together. Thus, in Problem 22, there are two decimal places in .02 and three in .015. So in the answer, 30, we must have $2 + 3$, or 5 places, making the result .00030. Of course the 0 at the right does not count and the final result is .0003.

16. 20×15 .

17. 200×15 .

18. 20×150 .

19. $2 \times .15$.

20. $2 \times .015$.

21. $.2 \times 15$.

22. $.02 \times .015$.

From these examples it is evident that the decimal point is not considered in operating the slide rule. After the work of the rule has been done, the decimal point is placed by looking at the problem and making a rough calculation.

23. 23×32 .

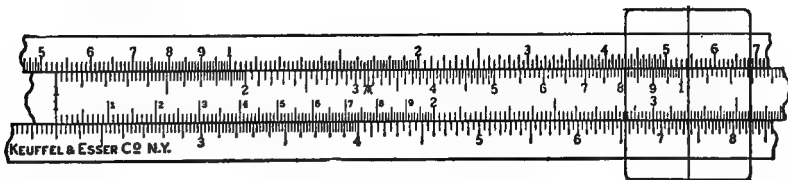
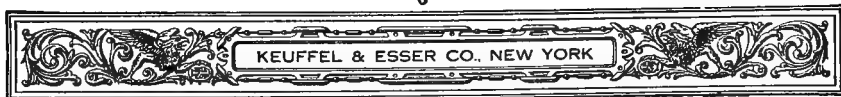


Fig. 8. $23 \times 32 = 736$.

Set 1 on *C* to 2.3 on *D*. Move the runner to 3.2 on *C*. Under the runner line on *D* find 736.

On the rule the hair line comes a little beyond 735, which is evidently 736. By looking at the example, you see that the units figure is 6 because $2 \times 3 = 6$.

If we had to depend entirely on the rule, the third figure might be 5, 6, or 7. So when we read numbers on the slide rule, we can be sure of the first two figures, and the third will not be more than one point away from the correct result.



PLACING THE DECIMAL POINT.

Since 23×32 is roughly 20×30 , or 600, place the decimal point in 736 so that the result will be as near 600 as possible. Evidently the answer is 736. (Fig. 8).

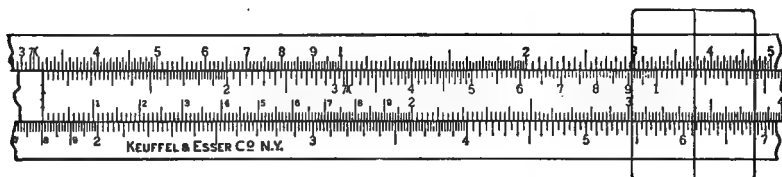


Fig. 9. $18 \times 34 = 612$.

24. 18×34 .

On the rule the result is a little over 61. Looking at the problem, we see that $4 \times 8 = 32$; hence 2 will be in the third place, making the order of figures 612.

By a rough calculation the problem is about equal to $20 \times 30 = 600$. Hence we make 612 look like 600 by placing the decimal point after the 2. The answer is 612.

25. 16×24 . (Answer, 384). 26. 14×26 . (Answer, 364).

27. Fill in the blanks in the following multiplication table, using the slide rule;

	31	32	33	34	35
21					
22					
23					
24					
25					
26					
27					
28					
29					
30					

WHICH INDEX TO USE.

We shall now call the 1 at the extreme left of the scale *C* the left-hand index, and the 1 at the extreme right of scale *C* the right-hand index.

In multiplying 30 by 34, if we use the left index of *C*, 34 comes beyond the right-hand end of the rule. In this case use the right-hand index, setting 10 on *C* to 30 on *D*.

The right or left-hand index must be used in accordance with the following rule: When the first digits of the factors are such that their product gives a number greater than ten, use the right index on *C*; otherwise use the left. Take as examples the following:

Example 1. 2.13×3.33 , $3 \times 2 = 6$. Use the left index.

Example 2. 7.23×4.71 , $7 \times 4 = 28$. Use the right index.

Example 3. $.131 \times 4.6$, $1 \times 4 = 4$. Use the left index.

If this very simple rule be kept in mind, much time will be saved. It will be a little difficult to make a decision in certain cases like the following: 3.13×3.31 . By the rule, the left-hand scale should be used, considering only the first digit of each of the factors; a closer inspection will show however, that this is not correct, and that the right index should have been used. The operator should not try to memorize these rules, but he should appreciate their significance.

28. Suppose you are earning 56 cents per hour and you are given an increase of 8 cents. What per cent increase do you receive?

Of course you will divide 8 by 56.

To divide one number by another on the slide rule we simply reverse the order of the work we have been doing in multiplication.

Set the runner to 8 on scale *D*.

Move the slide so as to set 56 on *C* to the hair line of the runner.

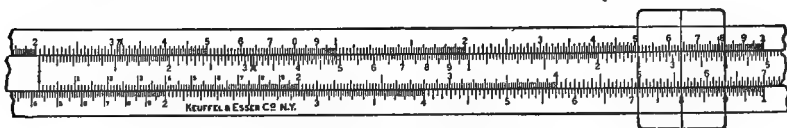


Fig. 10. $8 \div 56 = .14$.

Under 1 on *C* we find 14 and a little over. But the result is nearer 14 than 15. Hence the correct result to two figures is 14. By inspection the decimal point must be placed before the number, making the answer .14 or 14 per cent.

29. Before the war a man earned 35 cents per hour. While recovering from wounds, he learned a new trade which increased his earning power to 67 cents per hour. What per cent increase did he receive?

His increase is 32 cents per hour. The per cent of increase is found by dividing 32 by 35.

Set the runner to 32 on *D*.

Set 35 on *C* to the runner line. The result would be found under the 1 at the extreme left of scale *C*, but this 1 projects beyond scale *D*. So we use the 1 on the extreme right of scale *C*. Under this 1, find 91 on scale *D*. (Fig. 11).

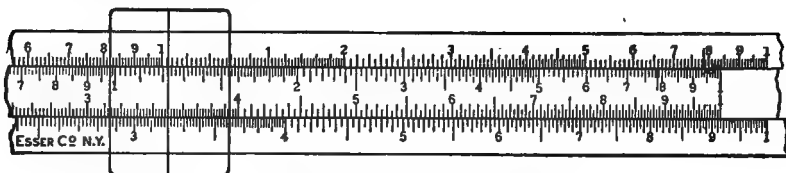


Fig. 11. $32 \div 35 = .91$.

In the same way, for practice, try the following:

30. What per cent of 91 is 45?

(Divide 45 by 91.)

31. What per cent of 73 is 24?

32. What per cent of 67 is 61?

33. What per cent of 53 is 31?

34. What per cent of 82 is 13?

35. What per cent of 42 is 9?

If you have a long report to make out in which a large number of per cents are to be calculated, why not use the slide rule?

A secretary to the president of a big corporation recently said: "The slide rule does my work in one-third of the time that would be required otherwise."

But suppose you had to get per cents in a problem like the following:

36. A baseball player made 57 hits out of 286 times at bat. What is his percentage?

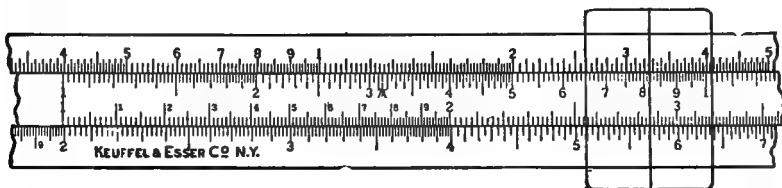


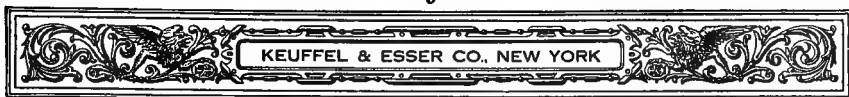
Fig. 12. $57 \div 286 = .199$.

Opposite 57 on *D* set 286 on *C*. When we look for 286 we observe that between 2.8 and 2.9 there are five spaces on the rule. Then every space counts one-fifth of .1, which is .02. Since we want six points for the third figure, we have to use three spaces, every one worth .02. $3 \times .02 = .06$.

Under the left-hand index find 199. When we read the result, we see that it comes on the rule between 1.9 and 2.0. There are ten small spaces between 1.9 and 2.0. Hence every space counts one point. We see that just a little practice in decimals enables us to read the rule to three figures. Now we must

place the decimal point. A rough calculation shows that $\frac{57}{286}$ is nearly $\frac{60}{300}$, or $\frac{1}{5}$.

Then the decimal point must be placed so as to make the result somewhere near one-fifth or .2. Evidently the result is .199. This may be read $19\frac{9}{10}$ per cent or $19\frac{9}{10}$ hundredths, or 199 thousandths.



37. If your income is \$2,500 per year and you save \$451, what per cent do you save?

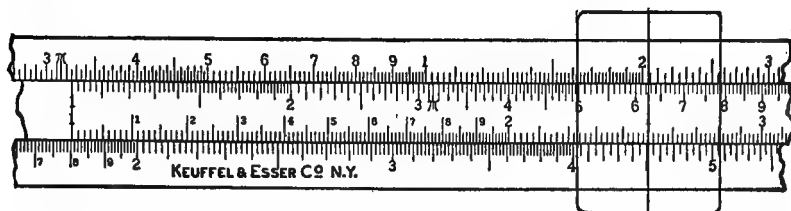


Fig. 13. $451 \div 2,500 = .180$.

Opposite 451 on *D* set 25 on *C*. Under the index find 180 on *D*. Hence the answer is .180, or 18 per cent. We note that when we look for the 1 in 451 on the rule, there are only two spaces between 45 and 46. Then each space counts one-half of a hundredth or one-half of .01, which is .005 or five points for the third figure. We estimate one-fifth of the small space to obtain .001.

38. If your salary is \$57.50 per week, and you are given an increase of \$12.40, what per cent increase do you receive?

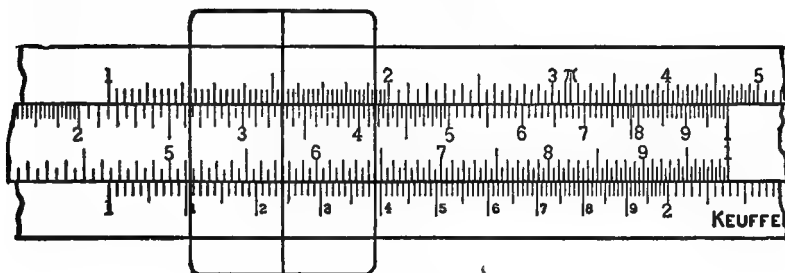


Fig. 14. $124 \div 575 = .216$.

Opposite 124 on *D* set 575 on *C*. This means that between 5 and 6 on *C* we must take 7 of the large divisions and one of the small divisions. Under the right-hand index read 216 on *D*. Hence the answer is $21\frac{6}{10}$ per cent.

39. $5.42 \div 2.42$.

40. $7.35 \div 3.14$.

41. $6.13 \div 4.61$.

42. $9.56 \div 7.26$.

43. $10 \div 3.14$.

For 10, use either the right or the left-hand index.

To place the decimal point these problems are roughly:

44. $16.5 \div .245$.

$16 \div .2 = 80$.

45. $.00655 \div .00034$.

$.0060 \div .0003 = 20$.

46. $.00156 \div 32.8$.

$.0015 \div 30 = .00005$.

47. $.375 \div .065$.

$.36 \div .06 = 6$.

48. $.0385 \div .0014$.

$.038 \div .001 = 38$.

Here is another method of placing the decimal point in division. Work the problem as though both dividend and divisor were integers (*i. e.*, not decimals), pointing off as usual. Move the decimal point to the left as many places as

there are decimal places in the dividend. Then move it to the right as many places as there are decimal places in the divisor. For example in problem 44, $165 \div 245$ gives .673. Move the point one place to the left because there is one decimal place in the dividend, giving .0673. Then move it three places to the right because there are three places in the divisor, giving as a result 67.3. Try both methods and see which one you like the better. Let one check the other.

Suppose we have more than three figures, as in the following example:
49. Find the circumference of a wheel 28 inches in diameter.

Here we must multiply 28 by 3.1416. But the slide rule only reads to three figures. So cut off the fourth and fifth figures in 3.1416 and call it 3.14, since the number is nearer 3.14 than 3.15.

50. Multiply 26 by 3.145.

In this case call 3.145 equal to 3.15.

51. If bell metal is made 25 parts of copper to 11 parts of tin, find the weight of tin in a bell weighing 404 pounds.

The tin is evidently eleven thirty-sixths of 404, or $\frac{11 \times 404}{36}$.

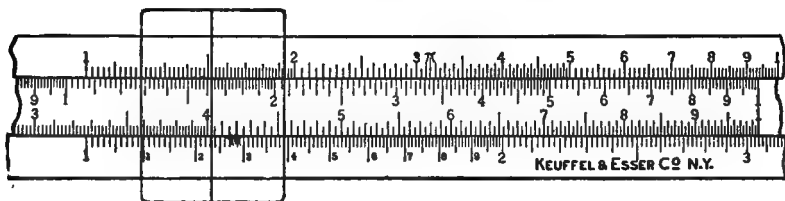


Fig. 15. $\frac{11 \times 404}{36} = 123.$

Opposite 11 on D set 36 on C. (Fig. 15).

Move the runner to 404 on C.

Under the runner line read 123 on D.

To place the decimal point, make a rough calculation as follows: The

example is roughly equal to $\frac{10 \times 400}{40} = 100$. So make 123 look as nearly like

100 as possible by placing the point after 3. The answer is 123 pounds of tin.

52. $\frac{14 \times 525}{47}$.

53. $\frac{24.5 \times 43.4}{3620}$.

54. $\frac{1.35 \times 3.16}{6.2}$.

(See Fig. 16).

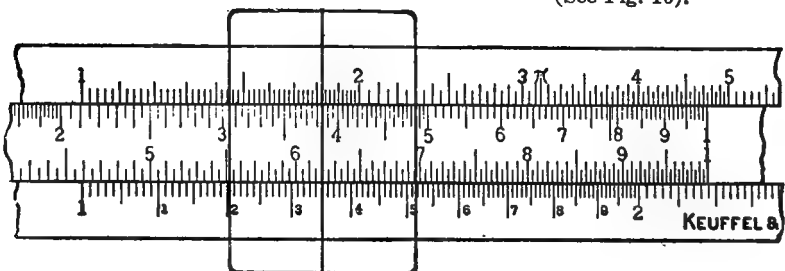
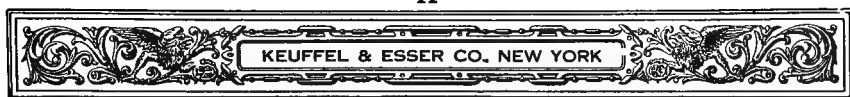


Fig. 16.



Opposite 1.35 on D, set 6.2 on C. If we try to move the runner to 316 on C, it is impossible because 316 lies beyond the extremity of D. In such a case proceed as follows: Move the runner to the right-hand index of C. (Fig. 17.)

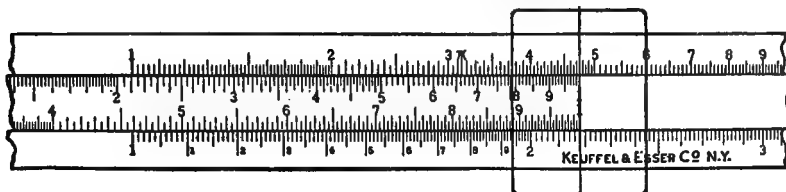


Fig. 17.

Then move the slide, setting the left-hand index of C to the runner line.

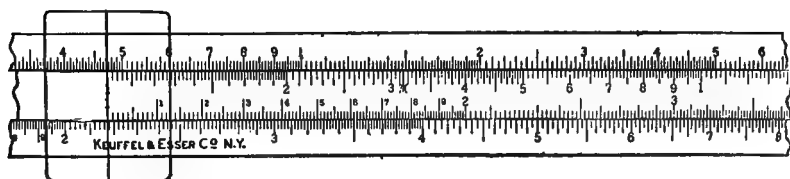


Fig. 18.

Now we can move the runner to 316 on C. (Fig. 19.)

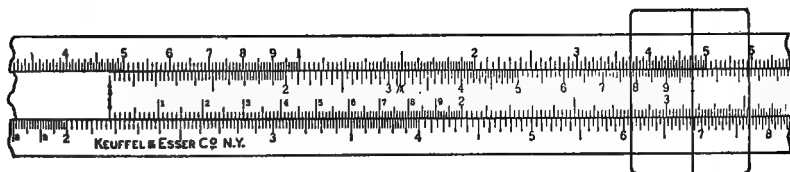


Fig. 19.

and read the result under the hair line on D. The answer is 688.

A rough calculation for the decimal point gives us $\frac{1 \times 3}{6} = \frac{3}{6}$, or .5. Making 688 look as much as possible like .5, we have .688.

$$\begin{array}{r} .55. \quad 2.28 \times .0125 \\ \hline 4.36 \end{array}$$

The rough calculation for the decimal point might be $\frac{2 \times .012}{4} = .006$.

$$\begin{array}{r} .56. \quad 7.63 \times 2.34 \\ \hline 24.3 \end{array}$$

$$\begin{array}{r} .57. \quad 2.56 \times 1.78 \\ \hline 7.4 \end{array}$$

$$\begin{array}{r} .58. \quad 82.5 \times 9.3 \\ \hline 56.5 \end{array}$$

$$\begin{array}{r} .59. \quad 32.6 \times 22.1 \\ \hline 9.25 \end{array}$$

60. If an aeroplane flying 100 miles an hour travels 86 miles in a given time, how far will an automobile traveling 22 miles an hour go in the same time?

Writing this in the form of a proportion:

$$100 : 86 = 22 : x$$

which means that 100 is to 86 as 22 is to the answer.

The work on the rule is as follows:

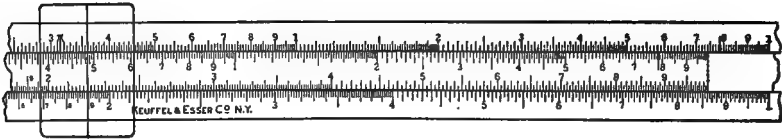


Fig. 20. $100 : 86 = 22 : 18.9$.

Opposite 86 on *D*, set 100 on *C*. (Use right index for 100). Move the runner to 22 on *C* and under the hair line on *D* read the answer, 18.9. An easy method of remembering this is:

$$\begin{array}{cccc} C & D & C & D \\ 100 : 86 = 22 : 18.9. \end{array}$$

In placing the decimal point, note that 100 has the same relation to 86 that 22 has to the answer. Since 86 is about nine-tenths of 100, we must place the decimal point in 189 so that the answer shall be about nine-tenths of 22. Hence the answer is 18.9.

In the same way solve the following proportions:

- 61. $24 : 31 = 15.2 : x$.
- 62. $1.4 : 2.5 = 12 : x$.
- 63. $3.71 : 2.4 = 51.2 : x$.
- 64. $2.54 : 4.72 = 7.48 : x$.
- 65. $12.3 : 15.2 = 8.5 : x$.

Problems from Camp Taylor (Field Artillery Training School):

- 66. If a post 13.2 feet high casts a shadow 27.2 feet long, how high is a tower which casts a shadow 116.8 feet long?

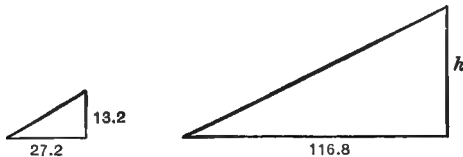


Fig. 21. $27.2 : 13.2 = 116.8 : h$.

$$27.2 : 13.2 = 116.8 : h$$

Solve like example 60.

- 67. At 2,400 yards an increase of 1 mil in the elevation of a gun increases the range 25.0 yards. What change in elevation will increase the range 40 yards?

The mil is the unit of angle in the artillery. It is equal to $\frac{1}{3438}$ of 360° .

- 68. The effects of wind on a shell are approximately proportional to the velocity of the wind. At 3,000 yards for a 3-inch gun, a rear wind of 10 miles per hour increases the range 30.1 yards. (a) What wind will increase the range 42.8 yards? (b) What wind will decrease the range 68.5 yards?

Answer: (a) Rear wind of 14.2 miles per hour. (b) Head wind of 22.8 miles per hour.

SQUARES.

69. Find the area of a square plot of ground measuring 128 yards on a side.

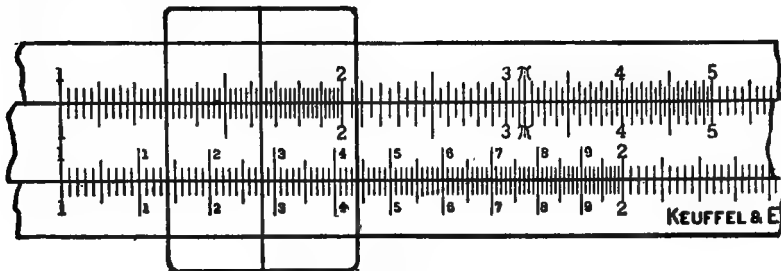


Fig. 22.

Set the runner to 128 on *D*. Directly above on *A* find the square required, 164. To place the decimal point, make a rough calculation.

$(128)^2$ is roughly $(130)^2$ or 16900. Then make 164 look like 16900 by placing the point as follows: 16400. The result is only correct to three figures. The complete result is 16384.

Case 1.—When the given number lies between 1 and 10, if the runner is set to this number on *D*, the square will be found opposite the runner line on *A*.

EXAMPLE:

70. Square 6.5.

The runner set to 6.5 on *D* indicates the square 42.25 on *A*. Notice that the *A* scale runs from 1 to 100 with 10 in the middle of the rule.

Case 2.—When the given number does not lie between 1 and 10, square as in case 1, and then place the decimal point by a rough calculation.

71. Example: Square 652.

Set the runner to 6.52 on *D* reading the square 425 on *A*. Roughly $(652)^2 = (600)^2 = 360,000$. To make 425 look as much as possible like 360,000, the decimal point must be placed as follows: 425,000. Notice that here the arithmetic square would be 425104, but on the slide rule we can only get the first three figures, 425. This, however, is close enough for most practical purposes, such as estimating on contract work.

Find the square of

72. 3.2	75. 8.65	78. .66	81. .0244
73. 4.65	76. 276.	79. .0625	82. 2240.
74. 1.12	77. 34.2	80. .0057	

83. Find the area of a circular plot of ground measuring 14.5 feet in diameter.

Use the formula $A = .7854 d^2$, which means that the area of the circle is equal to .7854 multiplied by the square of the diameter. Set the runner to 145 on *D*. The square is found directly above on *A*, but need not be read. Set the right-hand index of the slide to the runner line. Move the runner to .7854 on *B*. Opposite the runner line on *A* find the result, 165 sq. ft.

SQUARE ROOTS.

84. How long must one side of a square garden bed be made in order that it shall contain 8 square yards?

Here we have to find the square root of 8.

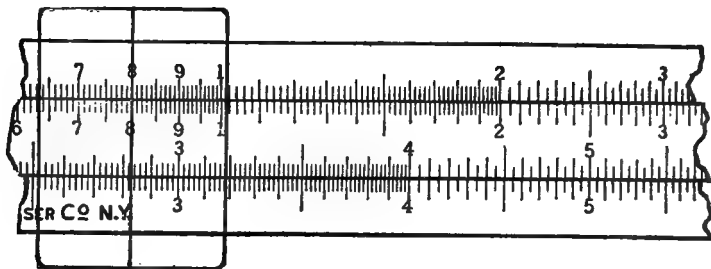


Fig. 23. $\sqrt{8} = 2.83$.

Set the runner to 8 on scale A. Note that scale A runs from 1 to 100, so that 8 is found on the left-hand half of the rule.

Now opposite the runner line on scale D, find 2.83, the square root. Then the result is 2.83 yards.

85. Find $\sqrt{3}$.

Set the runner to 3 on A.

Opposite the runner line find 1.73 on D.

86. Find $\sqrt{30}$.

Set the runner to 30 on A, being careful to notice that 30 is indicated by 3 on the right-hand half of the rule. Opposite the runner line on D, find 5.48.

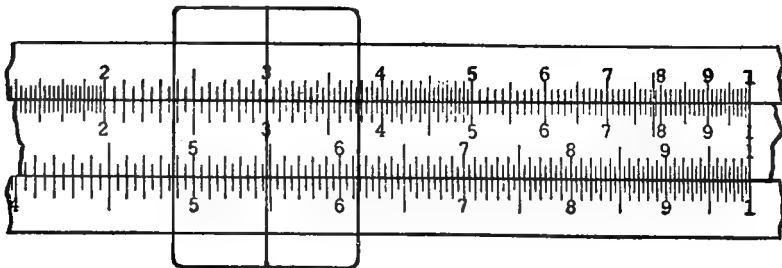


Fig. 24. $\sqrt{30} = 5.48$.

87. Find $\sqrt{300}$.

Since scale A only runs from 1 to 100, 300 is not found on the rule. Move the decimal point an even number of places in order to obtain a number that is on the rule. This can be done by moving the point two places to the left, giving $\sqrt{3.00}$.

Find the $\sqrt{3}$, which is 1.73. Then move the decimal point half as many places as it was moved in the first place, but in the opposite direction. In this case, move the point in 1.73 one place to the right, giving 17.3.

88. Find $\sqrt{30}$.

Move the point two places to the right, obtaining 30.

Find $\sqrt{30} = 5.48$.

Move the point one place to the left, obtaining .548 for the result.

89. Find $\sqrt{.03}$.

Move the decimal point two places to the right, obtaining $\sqrt{3}$.

Find $\sqrt{3} = 1.73$.

Move the point one place to the left, obtaining .173.

90. Find $\sqrt{.003}$.

Move the point four places to the right, obtaining $\sqrt{30}$.

Find $\sqrt{30} = 5.48$.

Move the point two places to the left, obtaining .0548.

Find the square roots of

91. 1.42. 94. .142. 96. 85.4. 98. .31416.

92. 14.2. 95. 2.43. 97. .365. 99. 1450.

93. 142.

100. Make a list of square roots of whole numbers between 110 and 130.

101. On a baseball field, find the distance from home plate to second base, measured in a straight line. (The distance between the bases is 90 feet).

102. Water is conducted into a tank through two lead pipes having diameters of $\frac{5}{8}$ and $1\frac{1}{4}$ inches, respectively. Find the size of the lead waste pipe that will allow the water to run out as fast as it runs in.

Use $\frac{5}{8}$ and $1\frac{1}{4}$ in the decimal form.

Find. $\sqrt{(.625)^2 + (1.75)^2}$.

NOTE:—Perform the addition by arithmetic. The slide cannot be used to advantage in addition.

103. Two branch iron sewer pipes, each 6 inches in diameter, empty into a third pipe. What should be the diameter of the third pipe in order to carry off the sewage?

WHEN MORE THAN TWO NUMBERS ARE TO BE MULTIPLIED TOGETHER.

104. Find the product of $3.2 \times 52 \times .24$.

Find 3.2×52 as usual, obtaining 166. Set the left-hand index to the runner line. Move the runner to 24 on C. Opposite the runner on D, read the answer, 39.9.

105. Find the product of $7.2 \times 3.2 \times .25 \times 5.4$.

7.2×32 gives 230. Set the left index to the runner. Move the runner to 25 on C. Opposite the runner on D, read 576. The product of the first three numbers is 57.6. Set the right index to the runner line. Move the runner to 5.4 on C. On D opposite the runner line, read 311. The decimal point is placed by a rough calculation.

The problem:

$7.2 \times 3.2 \times .25 \times 5.4$ is roughly equal to

$10 \times 3 \times .2 \times 5 = 30$.

Then 311 must be made to look as much as possible like 30. In order to do this the answer is evidently 31.1.

106. Find the product of $.75 \times 1.1 \times 6.5 \times 8.65$. Answer, 46.4.

107. Find the product of $8.2 \times .45 \times 6.4 \times 16$. Answer, 378.

108. Test for the simple operations of the slide rule.

TO THE STUDENT.

Read carefully the following instructions:

1. Copy the test on your paper in the form given below.
2. Work the problems straight through, setting down the answers in the column at the extreme right.
3. Fold these answers underneath the paper.
4. Work the problems through again, setting down the answers in the other column.
5. Compare the two sets of answers.
6. If the answers to any problem do not agree (within one point in the third place), work the problem again.
7. The correct results are given on page 71.

. TEST.

Credits			Answers Second Time	Answers First Time
20	1.	$1.28 \times 2.46.$		
20	2.	$84 \div 59.5.$		
20	3.	$58.5 \times 15.2.$ 78		
20	4.	$6.25 : 24.2 = 9.5 : x.$		
20	5.	$\sqrt{182}.$		

CHAPTER II.

THE THEORY OF THE SLIDE RULE.

This chapter includes a brief historical note on the Slide Rule, the theory of the operations described in Chapter I, and additional subjects, including Cubes, Cube Roots, Sines, Tangents and Logarithms.

HISTORICAL NOTE.

The earliest instrument that can be called a slide rule was invented by Wm. Oughtred (1574-1660), an English mathematician, and was described in publications of his pupil, Wm. Forster, in 1632 and 1653.

The present form of the rule was invented in 1859 by Lieutenant Amédée Mannheim, of the French Artillery. He had the good fortune to have his rule made by a firm of national reputation and to have it adopted by the French Artillery. In the United States the rule was little known until 1881, when Edwin Thacher invented the cylindrical form. Modern improvements have been introduced by Keuffel & Esser, such as celluloid facing, devices for smooth and even working of the slide in the stock, allowing for shrinking and warping, the addition of scales for cube root, sines, tangents and logarithms, inverted scales, the Duplex Form, and the 20-inch length.

For a complete history of the Logarithmic Slide Rule, the student is referred to "A History of the Logarithmic Slide Rule," by Florian Cajori, published by the Engineering News Publishing Company, New York City. This book traces the growth of the various forms of the rule from the time of its invention to 1909.

ACCURACY.

The accuracy of a result depends upon a), accuracy of the observed data, b), accuracy of mathematical constants, c), accuracy of physical constants.

ACCURACY OF THE OBSERVED DATA.

The precision of a measurement is evidently limited by the nature of the instrument and by the care used by the observer.

Example 1. If a distance is measured by a scale whose smallest subdivision is a millimeter, and the result recorded 134.8 mm., evidently the result is correct to 134, but the .8 is estimated. Hence it is known that the actual measurement lies between 134 and 135 and is estimated to be 134.8.

The result 134.8 is said to be "correct to four significant figures."

If the result were desired correct to only three figures, it would be recorded 135, since 134.8 is nearer 135.0 than 134.0. This result is said to be "correct to three significant figures."

Example 2. If the distance is measured by a rule whose smallest subdivision is .1 inch, and found to be exactly 8. inches, the result would be recorded 8.00 inches. The zeros record the fact that there are no tenths and no hundredths, but the distance is exactly 8 inches. The result, 8.00 inches, is said to be "correct to three significant figures."

Example 3. If an object is weighed on a balance capable of weighing to .01 gram, then .001 gram can be estimated. Suppose several objects are weighed with the following results:

1. Seven grams	recorded	7.000 grams.
2. Seven and a half grams	"	7.500 "
3. Seven and 9/100 grams	"	7.090 "
4. Seven and 6/1000 grams	"	7.006 "
5. 4/100 and 2/1000 grams	"	.042 "

Note that readings with the same instrument should show the same number of places filled in to the right of the decimal point, even if zero occurs in one or all of these places.

In number 5, the result, .042 grams is said to be "correct to two significant figures." The first significant figure is 4 and the second is 2.

Example 4. When we say that light travels 186,000 miles per second, we mean that the velocity of light is nearer 186,000 miles than it is 185,000 miles, or 187,000 miles. The result is said to be "correct to three significant figures."

Summarizing the preceding examples:

Example 1. 134.8 is correct to four significant figures.

Example 2. 8.00 is correct to three significant figures.

Example 3. .042 is correct to two significant figures.

Example 4. 186,000. is correct to three significant figures.

Counting from the left, the first significant figure is the first figure that is not zero.

After the first significant figure, zero may count as a significant figure, as in Example 2, where it represents an observed value, or it may not so count, as in Example 4, where the zeros merely serve to place the decimal point correctly, the number 186,000. being correct only to the nearest thousand miles.

ACCURACY OF MATHEMATICAL CONSTANTS.

A mathematical constant may be carried to any desired degree of accuracy, *e. g.*, the value of π usually given as 3.14159 has been calculated to 707 places. For ordinary calculations 3.14 or $3\frac{1}{2}$ is sufficiently accurate.

ACCURACY OF PHYSICAL CONSTANTS.

Most physical constants are only correct to three significant figures and some only to two figures.

e. g., The weight of a cu. ft. of water is 62.5 lb.

The weight of a cu. in. of cast iron is .26 lb.

LIMITS OF ACCURACY

HOLMAN'S RULE.

If numbers are to be multiplied or divided, a given percentage error in one of them will produce the same percentage error in the result.

In other words, a chain is no stronger than its weakest link.

Since physical constants are not usually correct beyond three significant figures, and the observed data in an experiment is rarely reliable beyond this point, the slide rule reading to three figures gives results sufficiently accurate for most kinds of practical work.

PERCENTAGE OF ERROR.

If a result is correct to three significant figures, the ratio of the error to the result is less than 1:100.

Suppose, for example, the result is 3527.6, which is known to be correct to three significant figures. Then the figures 352 are known to be correct and the figures 7.6 are doubtful.

Since 7.6 is less than 10 and 3527.6 is greater than 1000, the error must be less than 10:1000 or 1:100.

$$\frac{7.6}{3527.6} < \frac{10}{3527.6} < \frac{10}{1000}, \text{ or } \frac{1}{100}$$

A result read on the 10-inch slide rule to four significant figures is 1324, which is correct to three figures, 132, while the fourth figure, 4, is a close estimate not more than one point away from the correct reading.

The error here is less than $\frac{1}{1324}$, which is less than $\frac{1}{1000}$. Hence the error in this reading is less than one-tenth of one per cent.

By a similar method it may be shown that in a result correct to three significant figures, the error is less than one per cent. It will be found that readings on the left half of the slide rule may be obtained to four significant figures, or correct to one-tenth of one per cent.

On the right half of the rule where the spaces are smaller, readings may be obtained to three significant figures or correct to one per cent.

If greater accuracy is desired, a 20-inch rule will give one more significant figure and a Thacher Cylindrical Rule will give readings correct to five and sometimes six figures.

LOGARITHMS.

$$10^2 = 100.$$

Another form of making this statement is:

The logarithm of 100 is 2.

In the same way, $10^3 = 1,000$.

or the logarithm of 1,000 is 3.

From these examples it is evident that the logarithm is the exponent which is given to 10.

EXERCISE.

Fill out the blanks in the following table:

1. $10^4 = 10,000$	Log 10,000 =
2. $10^5 = 100,000$	Log 100,000 =
3. $10^1 = 10$	Log 10 =
4. $10^0 = 1$	Log 1 =

LAW OF MULTIPLICATION.

$$10^2 = 100.$$

$$10^3 = 1,000.$$

$$10^2 \times 10^3 = 100 \times 1,000.$$

$$10^5 = 100,000.$$

Log 100,000 is 5.

Since 5 is the sum of 2 and 3, $\log 100,000 = 2 + 3 = \log 100 + \log 1,000$, or

The logarithm of a product is the sum of the logarithms of the multiplicand and the multiplier.

Hence to multiply one number by another, add their logarithms.

The construction of the rule allows this addition to be done easily.

The scales are divided proportionally to the logarithms of the numbers.

If the 10-inch rule is considered as divided into 1,000 units, then any number; 1, 2, 3, etc., is placed on the rule so that its distance from the left index is proportional to its logarithm.

Since $\log 1 = 0$, 1 is found at the extreme left.

" $\log 2 = .301$, 2 is found 301 units from the left.

" $\log 3 = .477$, 3 " " 477 " " "

" $\log 9 = .954$, 9 " " 954 " " "

" $\log 10 = 1.000$, 10 " " 1000 " " "

On the scale the number 8 is placed three times as far from the left index as 2, because the logarithm of 8 is three times the logarithm of 2.

MULTIPLICATION.

When we multiply 2 by 4, we set the left index of the slide to 2 on scale *D*, and under 4 on scale *C* find the product, 8 on scale *D*.

This is equivalent to adding $\log 2$ to $\log 4$ and finding $\log 8$ (Fig. 25).

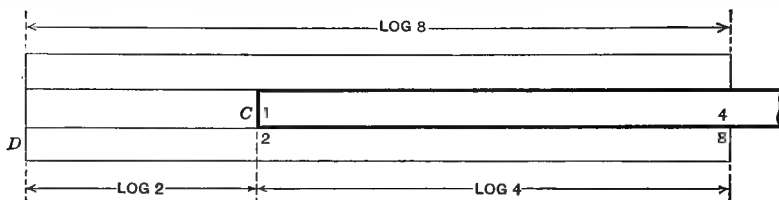


Fig. 25.

Example 1. Multiply 2.45 by 3.52.

Set 1 on *C* to 2.45 on *D*, and under 3.52 on *C* find 862 on *D*.

Roughly calculating, $2.45 \times 3.52 = 2 \times 4 = 8$.

Hence, we place the decimal point to make the result as near 8 as possible, and the result is 8.62.

Example 2. Multiply 24.5 by 35.2.

Working this like the preceding example, without regard to the decimal point, we obtain 862.

Roughly calculating, $24.5 \times 35.2 = 25 \times 36 = 900$.

Placing the decimal point to make 862 as near 900 as possible, we obtain 862.

Example 3. Multiply 6.234 by 142.6.

Taking each number correct to three significant figures we multiply 6.23 by 143.

Set 1 on *C* to 623 on *D*.

Under 143 on *C* find 885 on *D*.

Roughly calculating, $6 \times 140 = 840$.

Therefore the result is 885.

EXAMPLE 4. Multiply 2.46 by 7.82.

When the product of the first significant figures of each number is greater than 10, the sum of their logarithms will exceed the length of the rule. Hence if we set the left index of the slide to 246 on *D*, the other number 782 on *C* projects beyond the rule. In this case, think of the projection as wrapped around and inserted in the groove at the left, which would be the case in a circular slide rule. Now the right and left-hand indices coincide.

Hence set the right index of the slide to 246 on *D*.

Under 782 on *C* find 192 on *D*.

Roughly calculating, $2 \times 8 = 16$.

Hence the result is 19.2

EXAMPLE 5. Multiply .00146 by .0465.

Set 1 on *C* to 146 on *D*.

Under 465 on *C*, find 679 on *D*.

Roughly calculating, $.001 \times .05 = .00005$.

Hence the result is .0000679.

EXERCISE.

Find the value of

1. 2.34×3.16 .

4. 8.54×6.85 .

7. $.023 \times 2.35$.

2. 3.76×5.14 .

5. 34.2×7.55 .

8. $.00515 \times .324$

3. 1.82×4.15 .

6. 4.371×62.47 .

9. $.00523 \times .0174$.

10. Find the circumferences of circles having diameters of 4 ft., 6.5 ft., 14 ft.

Set 1 on *B*, to π on *A*.

Above 4, 6.5, and 14 read the circumferences on *A*.

DIVISION.

In division, reversing the operation of multiplication,

$$8 \div 4 = 2.$$

We subtract log 4 from log 8 and obtain log 2.

PROPORTION.

Problems in proportion are special cases of multiplication and division.

Example 1. Solve $16 : 27 = 17.5 : x$.

$$x = \frac{27 \times 17.5}{16}.$$

Solve by the method of Example 51, Chapter I.

First divide 27 by 16, by setting 16 on *C* to 27 on *D*. Then multiply by 17.5 by moving the runner line to 17.5 on *C*.

On *D*, opposite the runner line, will be found the result, 295.

To place the decimal point, note that 16 has the same relation to 27 that 17.5 has to x . Since 27 is not quite twice 16, x will be not quite twice 17.5. Hence the decimal point must be placed so that the answer is 29.5.

The method of working a proportion is easily remembered as follows:

$$\begin{array}{cc} C & D \end{array} \quad \begin{array}{cc} C & D \end{array}$$

$$16 : 27 = 17.5 : x.$$

Example 2. Solve $x : 24 = 11 : 18$.

$$\begin{array}{cc} C & D \end{array} \quad \begin{array}{cc} C & D \end{array}$$

$$x : 24 = 11 : 18.$$

To 18 on *D*, set 11 on *C*. Opposite 24 on *D* find x on *C*.

$$x = 14.7.$$

To place the decimal point, note that since 11 is a little more than half of 18, x will be a little more than half of 24, or 14.7.

SQUARES AND SQUARE ROOTS.

$$\begin{aligned} (10^3)^2 &= 10^3 \times 10^3 \\ &= 10^6. \end{aligned}$$

$$\text{Since } 6 = 2 \times 3,$$

$$\text{Log } (10^3)^2 = 2 \times \text{log } 10^3.$$

Hence, to square a number, multiply its logarithm by 2.

Scale *A* is graduated from 1 to 100, while scale *D* runs from 1 to 10.

Since the space given to each number on scale *D* is twice that on scale *A*, we multiply the log of a number by 2 if we look for it on scale *D*.

As an example, suppose we wish to square 3.

This can be done by doubling the space given to 3 on scale *A* and finding 9, or looking for 3 on scale *D* and finding its square above it on scale *A*.

Reversing the operation gives the square root.

As an example, find the square root of 9.

Look for 9 on scale *A*, and directly below it on *D* find 3, its square root.

CUBES.

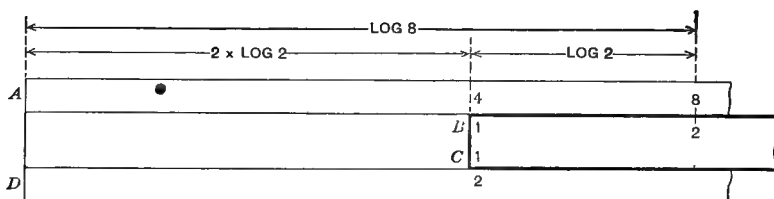


Fig. 26. $2^3 = 8$.

1. Find 2^3 .

Set the left index of the slide to 2 on *D*.

Opposite 2 on *B* read 8 on *A*.

Note that we have taken

$$2 \times \log 2 + \log 2 = 3 \log 2, \text{ or } \log 8$$

Scale *D*, Scale *B* Scale *A*

where $\log 2$ on *B* has been used as the unit.

In the same way, show that

2. $3^3 = 27$.

3. $4^3 = 64$.

4. $5^3 = 125$. Set right index on the slide to 5 on *D*.

5. $6^3 = 216$.

6. $7^3 = 343$.

7. $8^3 = 512$.

8. $9^3 = 729$.

9. $11^3 = 1,331$.

Roughly calculating, we know 11^3 is a little larger than 10^3 , or 1,000. We also can see that the units figure will be the cube of 1, or 1. The mark on the rule gives 133. Hence the total result is 1,331.

Show that

10. $12^3 = 1,728$.

Find the cubes of the following numbers correct to 4 significant figures.

Problem 11.	13.	Problem 16.	18.
12.	14.	17.	19.
13.	15.	18.	20.
14.	16.	19.	21.
15.	17.		

Find the cubes of the following numbers correct to 3 significant figures.

20.	22. (The complete answer is 10,648, but on the slide rule we get 10,600 correct to 3 significant figures. The error is less than one-half of one per cent.
21.	31.
22.	46.
23.	47. (Set the right-hand index on the slide to 47 on <i>D</i> .
24.	53.

25.	64.		
26.	758.	31.	.342.
27.	232.	32.	.057.
28.	425.6*	33.	.0068.
29.	87.9	34.	1.03.
30.	139.	35.	2.12.

*In Example 28, 425.6 is approximately 426. Roughly approximating the result $400^3 = 64,000,000$. The rule gives us 773. Hence the result is 77,300,000 correct to three significant figures. The complete result is 77,308,776.

Example 36. How many gallons will a cubical tank hold that measures 26 inches in depth. (1 gal. = 231 cu. in.).

CUBE ROOT.

Example 1. Find the cube root of 9.

Set the runner to 9 on scale *A*.

We know that the first figure of the root is 2, since the greatest cube in 9 is 8 and its cube root is 2.

Set the left index of the slide to 2 on scale *D*.

Opposite the index on *D* we have 2.00.

Opposite the runner line on *B* we have 2.25.

This first approximation tells us that the root lies between 2. and 2.25.

Next, try setting the index to 2.1 on *D*.

We now have:

Opposite the index on *D*, 2.10.

Opposite the runner line on *B*, 2.04.

This second approximation tells us that the root lies between 2.04 and 2.10; that is, correct to 2 places it is 2.0.

Setting the index in succession to numbers between 2.0 and 2.1, such as 2.02, 2.04, 2.06, 2.08, we find,

Opposite the index on *D*, 2.08.

Opposite the runner line on *B*, 2.08.

Check by cubing 2.08, observing that we already have the setting and 9 for the result.

Example 2. Find the cube root of 90.

Set the runner to 90 on scale *A*.

The greatest cube in 90 is 64 and its cube root is 4. Hence, the first significant figure is 4.

Set the left index to 4. on scale *D*.

Opposite the runner we have 5.6 on scale *B*.

The root lies between 4 and 5.6.

Set the index to 4.4 on *D*.

Opposite the runner we have 4.65 on *B*.

The root lies between 4.4 and 4.65.

Set the index to 4.48 on *D*.

Opposite the runner we have 4.48 on *B*.

The cube root is 4.48, correct to three figures.

Example 3. Find the cube root of 900.

Since the scale *A* is graduated only from 1 to 100, we cannot set the runner to 900.

Set it either to 9 or 90.

The greatest cube in 900 is 729 and its cube root is 9. Hence the first significant figure is 9.

If we set the left index to 9 on *D*, no part of the slide is opposite the runner. Hence, set the right-hand index of the slide opposite 9 on scale *D*.

This gives us:

On *D*, opposite the index, 9.

On *B*, opposite the runner line, 11.1.

The root lies between 9. and 11.1.

Set the index to 9.6 on *D*.

We now have:

On *D*, opposite the index, 9.6.

On *B*, opposite the runner line, 9.77.

The root lies between 9.6 and 9.77.

Set the index to 9.66 on *D*.

This gives us:

On *D*, opposite the index, 9.66.

On *B*, opposite the runner, 9.66.

Hence 9.66 is the root.

Solving arithmetically, the third figure is 5, but the fourth figure is more than 5, so that 9.66 is nearer the correct result than 9.65.

Example. 4. Find the cube root of .9.

We point off the number into periods of three figures each, counting from the decimal point, adding zeros to fill out the three figures.

This gives us .900.

Here we have the problem of the cube root of 900 repeated. From example 3, the significant figures are 966. In the cube root there is a decimal place for every decimal period in the given problem. Hence from the first decimal period .900 we obtain one decimal place, .9. This fixes the decimal point and the work on the rule gives us .966 for the root.

Example 5. Find the cube root of .09.

Following example 4, the first period is .090.

From example 2, the significant figures of the root are 448.

Hence, the result is .448.

From a consideration of these five examples, we have a rule for placing the decimal point in the cube root of numbers that do not lie between 1 and 1,000.

I. Move the decimal point 3, 6, or 9 places, as may be necessary, in either direction to obtain a number between 1 and 1,000.

II. Find the cube root of this new number.

III. In the result move the decimal point one third as many places as it was moved in 1, and in the opposite direction.

Example 6. Find the cube root of 56,342.

1. Move the decimal point three places to the left, obtaining 56.342.

2. Find the cube root of 56.3 which is 3.83.

3. Move the decimal point one place to the right, obtaining 38.3.

EXERCISE.

Find the cube roots of the following numbers:

1.	3.	11.	50.
2.	30.	12.	7.35
3.	300.	13.	.575
4.	.3	14.	241.
5.	.03	15.	3840.
6.	.003	16.	52076.
7.	2613.	17.	.0163
8.	47.8	18.	.0094
9.	.784	19.	1.036
10.	45083.	20.	108723.

21. How deep should a cubical box be made in order to contain 8,500 cubic inches?

CUBE ROOT BY THE INVERTED SLIDE.

Example 1. Find the cube root of 9.

Remove the slide from the groove, turn it through 180° in the horizontal plane, keeping the same face visible and insert it in the groove with scale *B* sliding next to scale *D*, and scale *C* next to scale *A*.

Set the right index of the slide to 9 on scale *A*.

Move the runner until you find the number on *D* that is opposite the same number on *B*. Note that 2.08 satisfies this condition and is the cube root as in Example 1, page 23.

An inspection of the rule will show that we have restored the condition of adding $2 \times \log 2.08$ on scale *D* to the $\log 2.08$ on *B*, giving the $\log 2.08$, or 9 on *A*.

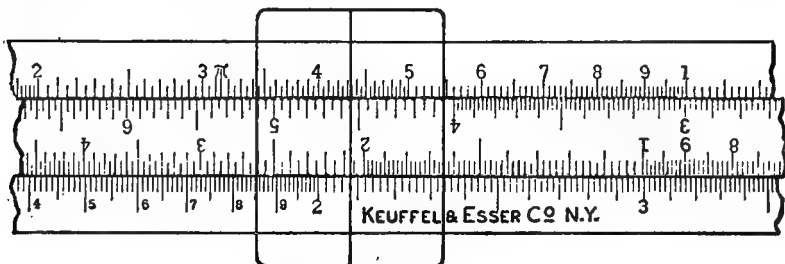


Fig. 27.

Try the problems in the above Exercise by this method.

As in Examples 1 to 5, page 23, the first significant figure should be calculated mentally.

Suppose we are finding the cube root of 900, using the inverted slide. If the left index is set to 9 on scale *A*, there are two numbers on scales *D* and *B* that coincide, 4.48 and 9.66. If the right index had been used, there would have been a third number, 2.08. Cubing 2.08, 4.48, or 9.66 gives the same sequence of figures 900, but the position of the decimal point differs. Mentally calculating the first significant figure before operating the rule determines which of these three numbers is the cube root desired.

MULTIPLICATION OF MORE THAN TWO NUMBERS.

Example 1. Find the value of $4.1 \times 56 \times .26 \times .49$.

1.—Using scales *C* and *D*, set the right index on *C* to 41 on *D* and move the runner to 56 on *C*. We have now multiplied 41 by 56. The result thus far found on *D*, opposite the runner line is 2296, without regard to the decimal point.

2.—Now set the left index of *C* to the runner line and move the runner to 26 on *C*, thus adding the log of 26 to the former result. On *D* under the runner line is 597.

3.—Set the right index to the runner line and move the runner to 49 on *C*. On *D* opposite the runner line, find 2925.

The position of the decimal point is determined by a rough calculation.

$4.1 \times 56 \times .26 \times .49$ is, roughly, $4 \times 60 \times \frac{1}{4} \times .5 = 30$.

Placing the decimal point so as to make 2925 read as near 30 as possible, it is evident that the result is 29.25.

EXERCISE.

Find the value of

1. $7.1 \times 31 \times .42$.

4. $8.25 \times .036 \times 1.07 \times 4.12$.

2. $.64 \times 32 \times 5.6$.

5. $6.2 \times 37.8 \times .0052 \times 46$.

3. $16.3 \times 1210 \times 3.65 \times 243$.

COMBINED MULTIPLICATION AND DIVISION.

Example 1. Find the value of

$$\frac{23.5 \times 45.3}{2670}$$

To 235 on *D*, set 267 on *C*. Move the runner to 453 on *C* and on *D*, opposite the runner line find 399. To obtain the decimal point make a rough calculation as follows:

$$\frac{23.5 \times 45.3}{2670} \text{ is roughly equal to } \frac{20 \times 50}{3000} = \frac{1}{3}.$$

Hence, we must place the decimal point so as to make 399 approximately equal to $\frac{1}{3}$. The result is evidently .399.

Another method of placing the decimal point:

$$\begin{aligned} \frac{23.5 \times 45.3}{2670} &= \frac{(2.35 \times 10) (4.53 \times 10)}{2.67 \times 1000} \\ &= \frac{2.35 \times 4.53}{2.67} \times \frac{1}{10} \\ &= 3.99 \times \frac{1}{10} \\ &= .399. \end{aligned}$$

The first method will be found preferable, but may be checked by the second.

Example 2. Find the value of $\frac{1.34 \times 2.15}{4.2}$.

To 1.34 on *D* set 4.2 on *C*. When we attempt to move the runner to 2.15 on *C*, it is impossible, because 2.15 projects beyond the left end of the rule. Bring the runner to 10 on *C* and move the slide so as to set the left index to the runner line. This divides by 10, but is permissible, since dividing by 10 does not change the order of significant figures. Now move the runner to 2.15 on *C* and on *D*, opposite the runner, read 686. A rough calculation shows that:

$$\frac{1.34 \times 2.15}{4.2} \text{ is approximately equal to } \frac{1 \times 2}{4} = \frac{1}{2}, \text{ or } .5.$$

Hence the result is .686.

Example 3. Find the value of

$$\begin{array}{r} 30.5 \times 50.6 \times 835 \\ \hline 3.64 \times 380 \times 42.5 \end{array} = x.$$

$$\begin{array}{r} \begin{array}{ccc} D & C & C \end{array} \\ 30.5 \times 50.6 \times 835 \\ \hline \begin{array}{ccc} 3.64 & 380 & 42.5 \\ C & C & C \end{array} \end{array} = x$$

The five operations are as follows:

1. At 305 on *D*, set 364 on *C*.
2. Move runner to 506 on *C*.
3. Set 380 on *C* to the runner line.
4. Runner to 835 on *C*.
5. Set 425 on *C* to the runner line, and opposite the index on *C* find 218 on *D*.

Intermediate Results on *D*.

838, opp. right index.
424, opp. runner.
111, opp. left index.
928, opp. runner.

Calculating roughly,

$$\frac{30 \times 50 \times 800}{3 \times 400 \times 40} = 25.$$

Hence 218 must be made to look as near as possible like 25, giving the result 21.8. It is not necessary to obtain the intermediate results, but with beginners it is an advantage to check the work at every step.

Example 4. Find the value of

$$\begin{array}{r} 25.4 \times 570 \times 26.8 \times 8.63 \times 1.3' \\ \hline 1.55 \times 8350 \times 4.15 \times 2.24 \\ \begin{array}{ccccccc} D & C & C & C & C & D \\ 25.4 \times 570 \times 26.8 \times 8.63 \times 1.3' \\ \hline \begin{array}{ccccccc} 1.55 & 8350 & 4.15 & 2.24 \\ C & C & C & C \end{array} \end{array} \end{array} = x.$$

Intermediates on *D*

- | | |
|---|-----|
| 1. At 254 on <i>D</i> , set 155 on <i>C</i> . | 164 |
| 2. Move runner to 570 on <i>C</i> . | 934 |
| 3. Move the slide, setting 835 to runner line. | 112 |
| 4. Runner to 268 on <i>C</i> . | 300 |
| 5. Move slide setting 415 on <i>C</i> to runner line. | 722 |
| 6. Runner to 863 on <i>C</i> . | 625 |
| 7. Move slide setting 224 to runner. | 279 |
| 8. Runner to 13 on <i>C</i> . | 363 |

Find the answer 363 on *D* opposite the runner.

Calculating roughly:

$$\frac{25 \times 600 \times 30 \times 8 \times 1}{1 \times 8000 \times 4 \times 2} = 60.$$

Making 363 look as much as possible like 60 we have 36.3.

Example 5. Find the value of

$$\frac{7.45}{3.65 \times .0267}$$

Examples 1 to 4 have had as many factors in the numerator as in the denominator or one more. This example can be changed to conform to these types by introducing one as a factor in the numerator.

$$\frac{7.45}{3.65 \times .0267} = \frac{\overset{D}{7.45} \times \overset{C}{1}}{\underset{C}{3.65} \times \underset{C}{.0267}} = x.$$

Check by
Intermediates on *D*.

- | | |
|---------------------------------------|----------------------------|
| 1. Divide 7.45 by 3.65. | 204, Opposite left index. |
| 2. Move runner to 1 on <i>C</i> . | 204, Opposite runner. |
| 3. Move slide, setting 267 to runner. | 764, Opposite right index. |

Roughly calculating:

$$\frac{8 \times 1}{4 \times .02} = \frac{2.00}{.02} = 100.$$

Making 764 look as much as possible like 100, the result is 76.4.

Example 6.

Find the value of $\frac{1}{2.34 \times .33 \times 5.25}$.

Writing the example in type form we have:

$$\frac{\overset{D}{1} \times \overset{C}{1} \times \overset{C}{1}}{\underset{C}{2.34} \times \underset{C}{.33} \times \underset{C}{5.25}} = x.$$

Check by
Intermediates on *D*

- | | |
|---|-----------------------------|
| 1. At 1 on <i>D</i> , set 234 on <i>C</i> . | 427, opposite right index. |
| 2. Runner to 1 (right index) on <i>C</i> . | 427, opposite runner. |
| 3. 33 on <i>C</i> to runner. | 1295, opposite left index. |
| 4. Runner to 1 (left index) on <i>C</i> . | 1295, opposite runner. |
| 5. 525 on <i>C</i> to runner. | 2467, opposite right index. |

Rough calculation:

$$\frac{1}{2 \times \frac{1}{3} \times 6} = \frac{1}{4} = .25.$$

Making 2467 look as much as possible like .25 the result is .2467.

Example 7.

Find the value of: $\sqrt{\frac{21.4 \times 3.45 \times 640}{4.15 \times .75 \times .08}}$.

FIRST METHOD. —Work the example without regard to the square root, then find the square root of the result:

SECOND METHOD.—Using scales *A* and *B*:

$$\begin{array}{cccc} A & B & B & D \\ 21.4 \times 3.45 \times 640 & & & \\ \hline 4.15 \times .75 \times .08 & & & \\ B & B & B & \end{array} = x.$$

Intermediate on *A*.

516, opposite index.

1. To 21.4 on *A* set 4.15 on *B*.
Be careful to use 21.4 on the right half of *A* and not 2.14 on the left half, since the square root of 21.4 has different significant figures from the square root of 2.14. For the same reason use 4.15 on the left half of *B*.

2. Runner to 3.45 on *B* (left half of rule) 178, opposite runner.

3. Move slide setting .75 (right half) to runner. 237, opposite index.

4. Runner to 6.4 (left half) on *B*. 152, opposite runner.

Change 640 to 6.4, by moving the decimal point an even number of places, in order not to change the square root.

5. Move slide setting 8 (left half) on *B* to runner. 190, opposite index.

6. Opposite right index of *B* find 436 on *D*.

Without taking the square root the result is 190 on *A*. Dropping down to *D* gives the square root, 436.

Rough Calculation

$$\sqrt{\frac{20 \times 3 \times 600}{4 \times 1 \times .1}} = \sqrt{90000} = 300.$$

Placing the decimal point so as to make 436 as near as possible to 300, the result is 436.

EXERCISE.

Find the value of:

1. $\frac{3.26 \times .0235}{4.22}.$

2. $\frac{6.75 \times 1.35}{14.4}.$

3. $26.4 \times 4.8 \times 7.12.$

4. $6.2 \times 28 \times .35 \times 5.4.$

5. $\frac{.65 \times 24 \times 7.5 \times 9.5}{6.45}.$

6. $\frac{4.55 \times .0276}{1}.$

7. $\frac{1}{2.66 \times .75 \times 1.42}.$

8. $\frac{3}{2.54 \times 7.45}$

9. $\frac{2.14 \times 4.6 \times .39}{24.3 \times .06 \times .575}$.
10. $\frac{5.8 \times 4.5 \times 8.7 \times 132}{7.3 \times 6.2 \times 28 \times 14}$.
11. $\frac{2.63 \times 82.5}{2450}$.
12. $\frac{48.6 \times 22.4}{56.5 \times 245}$.
13. $\frac{22.5 \times 12.2 \times 126 \times 405}{2760 \times 715 \times 6.16}$.

MISCELLANEOUS CALCULATIONS.

Find the value of:

$$\frac{2.45 \times (76.5)^2 \times 625}{.55 \times .087}$$

METHOD I. Use scales *A* and *B*, but use *C* for 76.5.

At 245 on *A* set 55 on *B*.

Runner to 765 on *C*.

Set 87 on *B* to the runner.

Runner to 625 on *B*.

Opposite the runner on *A*, find 187.

A rough calculation shows:

$$\frac{2 \times 70 \times 80 \times 600}{.5 \times .1} = \frac{200 \times 70 \times 80 \times 600}{5 \times 1} = 134000000.$$

The result is 187,000,000.

METHOD II. Write the example:

$$\frac{2.45 \times 76.5 \times 76.5 \times 625}{.55 \times .087 \times 1}$$

METHOD III. Find $(76.5)^2$ as a separate problem, then work the example on *C* and *D*.

Example 2. Find the value of:

$$\frac{135 \times \sqrt{475} \times 430}{26 \times 250 \times 638}$$

Use *C* and *D*, but use *B* for 475.

At 135 on *D*, set 26 on *C*.

Runner to 4.75 on *B* (left half of slide, because the decimal point must be moved an even number of places).

Set 250 to the runner.

Runner to 430 on *C*.

Set 638 to the runner.

On *D*, opposite the right-hand index, find 305.



Roughly calculating:

$$\frac{100 \times 20 \times 400}{25 \times 250 \times 600} = \frac{16}{75} = \text{about } 1/5, \text{ or } .2.$$

The result, then, is .305.

Example 3. Find the value of

$$\frac{\sqrt{260} \times \sqrt{3.8}}{\sqrt{1310}}.$$

Use *A* and *B*, but read the result on *D*.

At 2.6 on *A* set 13.1 on *B* (moving the decimal point an even number of places).

If we try to move the runner to 3.8 on *B*, 3.8 projects beyond the end of the rule. Hence, move the runner to the right index of the slide, then set the left index to the runner. This operation divides by 100, but does not change the significant figures of the result.

Now move the runner to 3.8 on *B*.

On *D*, opposite the runner, read 869.

Roughly calculating:

$$\sqrt{\frac{300 \times 3}{1600}} = \frac{3}{4} = .75.$$

Hence the result is .869.

1. $\frac{13.5 \times (14)^2}{82}$

2. $1.35 \times \sqrt{2.}$

3. $\frac{42.3}{\sqrt{6720}}$

4. $\frac{5.2}{(3.4)^2}$

Settings:

The Result is Denoted by *x*

A : To 135 : Find *x*

B : Set 82 :

C : : Over 14

D :

A : : :

B : : Under 2

C : Set 1 :

D : To 135 : Find *x*

A : : :

B : Set 67.2 :

C : : Under 10

D : To 42.3 : Find *x*

A : To 5.2 : Find *x*

B : : Over 1

C : Set 3.4 :

D : : :

$$5. \frac{(16.2)^2 \times 45.2}{(2.7)^2}.$$

$$6. \left\{ \frac{.0347}{.0058} \right\}^2$$

$$7. \frac{2.31 \times (48.5)^2 \times 413}{.45 \times .087}.$$

$$8. \frac{175 \times \sqrt{285} \times \sqrt{17} \times 410}{28 \times 228 \times 634}.$$

$$9. \frac{\sqrt{8.32} \times \sqrt{56.5}}{\sqrt{2830}}.$$

$$10. \frac{2.6}{(7.4)^3}.$$

$$11. \left\{ \frac{.0325}{.0075} \right\}^3.$$

$$12. \sqrt[3]{\frac{420 \times 1.65}{2.64}}.$$

A : : Find *x*

B : : Over 452

C : Set 27 :

D : To 162 :

A : : Find *x*

B : : Over 100

C : Set 58 :

D : At 347 :

THE INVERTED SLIDE.

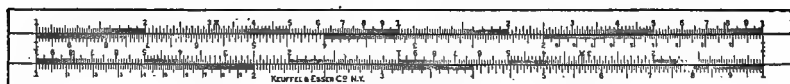


Fig. 28.

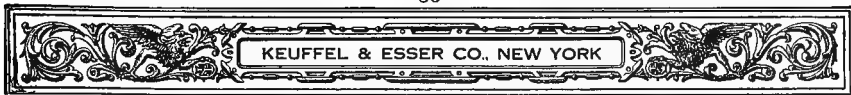
Remove the slide from the groove, keeping the same face uppermost, turn it through 180 degrees, and insert it in the groove. This will bring scale *B* adjacent to scale *D* and scale *C* adjacent to scale *A*, as in Fig. 28. We call this "inverting the slide." The scales on the slide we will now call *CI* (*C* inverted) and *BI* (*B* inverted).

From the figure, $\log 2 + \log 5 = \log 10$.

Hence, $2 \times 5 = 10$.

Or $2 = .5 = 1$.

In the same way, it is evident that any number on scale *D* multiplied by the number above it on scale *CI*, is equal to 10., because the sum of the logs of these numbers extends the entire length of the rule.



e. g., $D \quad CI$
 $4 \times 2.5 = 10.$
 and $4 \times .25 = 1.$

RECIPROCALLS.

Two numbers are reciprocals if their product is equal to unity., *e. g.*, 2 is the reciprocal of .5.

To find the reciprocal of a number, invert the slide, set 10 on the slide to 1 on *D*, and every number on *D* has for its reciprocal one-tenth of the number above it on *CI*.

EXERCISE.

1. Using the inverted slide, make a table of the reciprocals of the integers from 1 to 10.
2. Make a table of the reciprocals of the numbers 1.4, 1.5, 1.6, etc., to 2.4, inclusive.
3. Change $1/32$ inch to a decimal.

Solution:—

Let $x = 1/32.$

Then $x \times 32 = 1.$

Or x is the reciprocal of 32.

On the rule, above 32 on *D*, find 3125 on *CI*.

Placing the decimal point by inspection, $x = .03125.$

4. Express $1/64$ inch as a decimal.

MULTIPLICATION.

Example 1. Multiply 3 by 2, using the inverted slide.

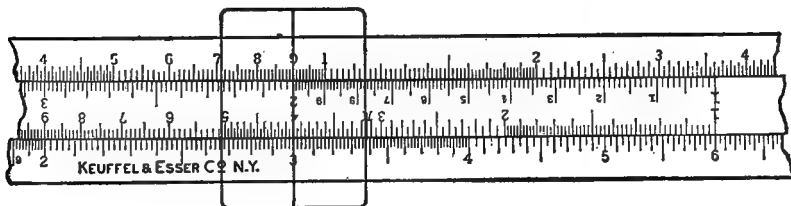


Fig. 29.

Set the runner to 3 on *D*.

Set 2 on *CI* to the runner.

Under 1 on *CI*, find 6 on *D*.

EXPLANATION. Evidently we have added $\log 3$ on *D* to $\log 2$ on *CI*, and the sum is $\log 6$, found on *D*.

The advantage of using the inverted scale in multiplication is that there is never any uncertainty as to which index to use.

The disadvantage is that scales *CI* and *D* are separated, making it more difficult to set one number opposite another.

EXERCISE.

Work the examples in Chapter II, Page 20 by the inverted slide, comparing this with the method by the direct slide.

Example 2. Find the value of $6 \times 2 \times 8$, using the inverted slide.

Set the runner to 6 on *D*.



Set 2 on *CI* to the runner.

Move the runner to 10 on *CI*.

Set 8 on *CI* to the runner.

Opposite 1 on *CI*, find 96 on *D*.

Example 3. Find the value of $1.25 \times 1.42 \times 6.14$ by the inverted slide.

Set the runner to 125 on *D*.

Set 142 on *CI* to the runner.

Runner to 1 on *CI*.

Set 614 on *CI* to the runner.

Under 10 on *CI*, find 109 on *D*.

Placing the decimal point by inspection, the result is 10.9.

EXERCISE.

Find the value of

1. $6.1 \times 24 \times .32$.

2. $.53 \times 42 \times 6.5$.

3. $14.3 \times 126 \times .238$.

4. $6.25 \times .014 \times 58$.

5. $7.4 \times 42.5 \times .85$.

DIVISION.

Example 1. Divide 6 by 2, using the inverted slide.

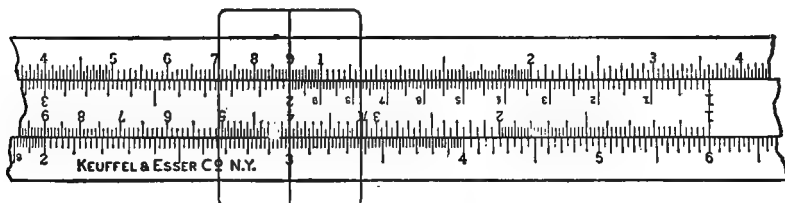


Fig. 30.

To 6 on *D*, set 1 on *CI*.

Under 2 on *CI*, find 3 on *D*.

EXPLANATION. We have subtracted $\log 2$ on *CI* from $\log 6$ on *D*, and obtained $\log 3$ on *D*.

Example 2. Divide 3.72 by 14.5.

To 372 on *D*, set 1 on *CI*.

Under 145 on *CI*, find 256 on *D*.

Roughly calculating, $3.72 \div 14.5 = 3/15 = .2$.

Hence, the result is .256.

Example 3. Divide 12.5 by 6.8.

If we set 1 on *CI* to 125 on *D*, 6.8 is beyond the left end of the rule. We therefore use the other index.

To 125 on *D*, set 10 on *CI*.

Under 68 on *CI*, find 184 on *D*.

Placing the decimal point by inspection, the result is 1.84.

EXERCISE.

Find the value of the examples on Page 9, using the inverted slide.

SINES.

FIRST METHOD.

Remove the slide from the groove, turn it over so that the face that was underneath is now uppermost and insert it in the groove with the indices coinciding as in Fig. 31.



Fig. 31.

The scale marked *S* is a scale of sines. Angles are given on scale *S*, opposite their sines on scale *A*.

Example 1. Find $\sin 20^\circ$.

Opposite 20 on scale *S* is found its sine on scale *A*. This reads 342. To place the decimal point, a number read on the right half of scale *A* has the first significant figure in the first decimal place, except sine 90, which is 1; a number read on the left half of scale *A* has the first significant figure in the second decimal place.

Hence $\sin 20^\circ = .3420$.

Example 2. Find $\sin 2^\circ$.

The significant figures are 349.

The reading is on the left of scale *A*, hence the result is .0349.

SECOND METHOD.

Without reversing the slide, set the given angle on scale *S* to the mark opposite the right index on the under side of the rule; then under the right index of scale *A* read the sine on scale *B*.

COSINES.

Since the cosine of an angle is equal to the sine of the complement of the angle, the cosine may be found on the slide rule.

Example. Find $\cos 30^\circ$.

$$\cos 30^\circ = \sin (90^\circ - 30^\circ).$$

$$= \sin 60^\circ.$$

$$= .866.$$

Example 3. Find $\sin 5^\circ 40' \times 35$.

METHOD I.—With the slide having scales *B* and *C* uppermost, set $\sin 5^\circ 40'$ on *S* to the mark in the hollow at the right end of the rule.

Under 35 on *A*, read the product 3.46 on *B*.

EXPLANATION.

Evidently we have added $\log \sin 5^\circ 40'$ to $\log 35$, the sum being counted on scale *B*.

Or $\sin 5^\circ 40' \times 35 = x$ may be written as a proportion using scales *A* and *B*.

$$\begin{array}{cccc} A & B & A & B \\ 1 : \sin 5^\circ 40' = 35 : x. \end{array}$$

Under 1 on *A*, set sin 5° 40' on *B*.

Under 35 on *A*, find 3.46 on *B*.

METHOD II.—With the slide having scales *S* and *T* uppermost,

A : to 35 : Find 3.46

S : Set Right Index : Over 5° 40'

EXPLANATION.

Log 35 is added to log sin 5° 40', the sum being counted on scale *A*.

Example 4. Find $\frac{35}{\sin 5^\circ 40'}$.

METHOD I.—With the slide having scales *A* and *B* uppermost, set sin 5° 40' to the mark in the hollow at the right end of the rule.

Over 35 on *B*, read the quotient 354 on *A*.

METHOD II. With scale *S* uppermost

A : To 35 : Find 354.

S : Set 5° 40' : Over left index.

To place the decimal point, note that sin 5° 40' is a trifle less than .1. Hence dividing 35 by .1 we have 350 for the rough calculation.

EXPLANATION.

METHOD I. We have solved the proportion:

$$\sin 5^\circ 40' : 1 = 35 : x.$$

B *A* *B* *A*.

METHOD II.—We have taken the proportion by alternation securing.

$$\sin 5^\circ 40' : 35 = 1 : x.$$

S *A* *S* *A*.

EXERCISE.

Find the sines of:

1. 90°.	6. 15° 20'.
2. 45°.	7. 1° 30'.
3. 30°.	8. 8° 30'.
4. 3°.	9. 2° 15'.
5. 40'.	10. 21° 30'.

Find the cosine of:

11. 80°.	16. 75° 30'.
12. 65°.	17. 54° 10'.
13. 42°.	18. 20° 30'.
14. 14°.	19. 81° 45'.
15. 12°.	20. 88° 25'.
21. Sin 25° × 45.	
22. Cos 56° × 27.	

23. $\frac{18}{\sin 12^\circ 30'}$.

24. $\frac{21.5}{\sin 42^\circ 10'}$.

25. $A = 32^\circ$.
 $c = 65$
 Find a
26. $A = 70^\circ 30'$.
 $a = 15.4$.
 Find c .

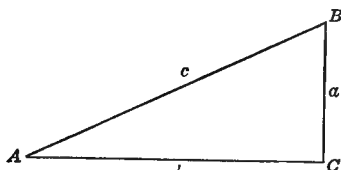


Fig. 32

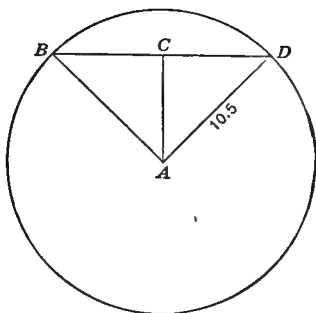


Fig. 33.

27. A disk is 21 inches in diameter.
 Find the distance necessary to set a pair of dividers in order to space off a), 7 sides; b), 8 sides; c), 10 sides; d), 13 sides.

The angle $DAB = 1/7$ of $360^\circ = 51^\circ 26'$
 (to the nearest minute). f'

The angle $DAC = 1/2$ of $51^\circ 26' = 25^\circ 43'$.

$\frac{CD}{AD} = \text{sine angle } DAC$.

$CD = AD \times \text{sine angle } DAC$.

$BD = 2 \times CD$.

28. Holes A and C are to be drilled on the milling machine. After drilling C, in order to drill A, how much movement of the table will there be in each direction?

The table moves from C to B, then from B to A.

$$BC = 5 \times \cos 20^\circ.$$

$$BA = 5 \times \sin 20^\circ.$$

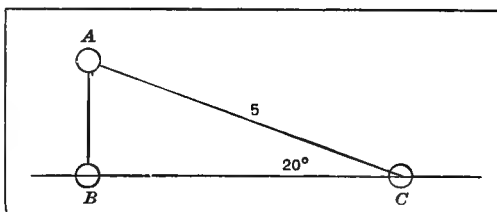


Fig. 34.

TANGENTS.



Fig. 35.

With the slide in position for reading sines, scale T gives readings for angles whose tangents are found opposite on scale D .

The first significant figure comes in the first decimal place for all values found on the rule.

Example 1. Find $\tan 30^\circ$.

METHOD I. Opposite 30 on scale *T*, find 577 on *D*.

Pointing off, we have $\tan 30^\circ = .5770$, which is correct to three significant figures, the result correct to four figures being .5774.

METHOD II.

With the slide in the usual position showing scales *B* and *C*, set 30 on the *T* scale to the mark in the notch at the left end of the rule and opposite 1 on *D* read 577 on *C*.

Example 2. Find the value of $\tan 18^\circ 30' \times 175$.

METHOD I.—Find $\tan 18^\circ 30'$ by Example 1, Method II.

Above 175 on *D*, find 586 on *C*.

Since $\tan 18^\circ 30'$ is .3346, the product must be roughly $1/3$ of 175, making the result 58.6.

The scale gives tangents only as far as 45° .

For larger angles, use the formula:

$$\tan A = \frac{1}{\tan (90^\circ - A)}$$

Example. Find the $\tan$ of 75° .

$$\begin{aligned}\tan 75^\circ &= \frac{1}{\tan (90 - 75)} \\ &= \frac{1}{\tan 15^\circ} \\ &= \frac{1}{.2679} \\ &= 3.73.\end{aligned}$$

Example 3. Find the value of $565 \div \tan 65^\circ$.

$$\begin{aligned}565 \div \tan. 65^\circ &= 565 \div \frac{1}{\tan 25^\circ} \\ &= 565 \times \tan 25^\circ \\ &= 263.\end{aligned}$$

Example 4. Find the value of

$$256 \div \tan 10^\circ 30'.$$

METHOD I.—To 256 on *D* set $10^\circ 30'$ on *T*. Under the left index on *T*, find 138 on *D*.

By a rough calculation, remembering $\tan 45^\circ = 1$,

$$\frac{256}{\tan 10^\circ 30'} = \frac{256}{.18} = 172.$$

Making 138 look as much as possible like 172, we have 138.

METHOD II.—With the *C* and *D* scales uppermost, set $10^\circ 30'$ on *T* to the mark in the groove at the left end of the rule.

Under 256 on *C*, find 138 on *D*.

Example 5. Find the value of

$$256 \div \tan 40^\circ 10'.$$

By Method II, setting $40^\circ 10'$ on *T* to the mark in the groove at the left end of the rule, we find 256 on *C* comes beyond the left end of the rule.

Set the runner to 10 on *C*, then set 1 on *C* to the runner.
Now, under 256 on *C*, find 303 on *D*.

$$\text{Roughly calculating: } \frac{256}{\tan 40^\circ 10'} = \frac{256}{8} = 320.$$

Hence, the result is 303.

The tangent of an angle less than $5^\circ 43'$ cannot be obtained directly from the ordinary 10 in. rule, but the sine may be used in place of the tangent, since the sine and the tangent of any of these angles are identical to three significant figures.

$$\tan 1^\circ 30' = \sin 1^\circ 30' = .0262.$$

COTANGENTS.

The cotangent may be found as follows:

$$\text{Cot } A = \tan (90^\circ - A).$$

Example: Find $\cot 65^\circ$.

$$\begin{aligned}\text{Cot } 65^\circ &= \tan (90^\circ - 65^\circ) \\ &= \tan 25^\circ \\ &= .466.\end{aligned}$$

SECANT AND COSECANT.

The secant and cosecant may be found by the formulas:

$$\sec A = \frac{1}{\cos A}.$$

$$\csc A = \frac{1}{\sin A}.$$

EXERCISE.

Find the tangents of the following angles

- | | | | |
|----|------------------|-----|------------------|
| 1. | 25° . | 7. | $75^\circ 10'$. |
| 2. | $14^\circ 30'$. | 8. | $20^\circ 10'$. |
| 3. | $35^\circ 30'$. | 9. | $15^\circ 5'$. |
| 4. | $26^\circ 20'$. | 10. | $6^\circ 25'$. |
| 5. | $18^\circ 30'$. | 11. | $1^\circ 45'$. |
| 6. | $55^\circ 20'$. | 12. | $42^\circ 20'$. |

13. $\tan 15^\circ \times 18$.

14. $\tan 65^\circ 30' \times 13.2 = \frac{13.2}{\tan 24^\circ 30'}$.

15. $\frac{5.62}{\tan 10^\circ}$.

16. $\frac{8.5}{\tan 70^\circ 20'} = 8.5 \times \tan 19^\circ 40'$.

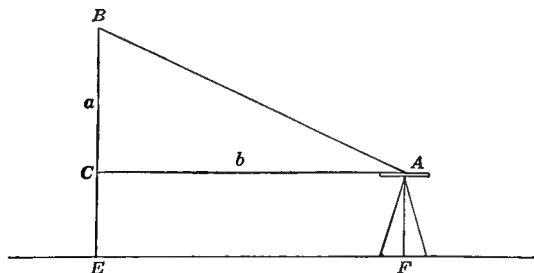


Fig. 36.

17. To find BE, the height of a building, a transit is set up at A, a level line AC is sighted on a rod held at E.

CE is found to be 5.2 ft.

EF, which is equal to CA, is measured and found to be 138 ft.

The angle CAB is taken by the transit and found to be $28^{\circ} 30'$.

Find BE, the height of the building.

$$\begin{aligned} BE &= BC + CE. \\ BC &= CA \times \tan A. \\ BE &= CA \times \tan A + CE. \\ &= 138 \times \tan 28^{\circ} 30' + 5.2. \\ &= 74.9 + 5.2. \\ &= 80.1 \text{ ft.} \end{aligned}$$

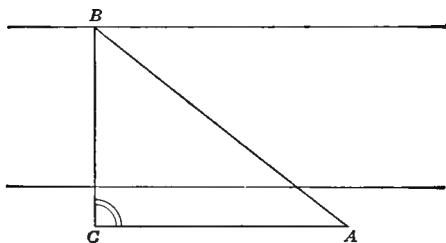


Fig. 37.

18. To find CB, the width of a river.

A transit is set up at C and a right angle, BCA is laid off.

CA is measured and found to be 235 ft.

Then the transit is set up at A and the angle A found to be $75^{\circ} 30'$.

Find CB, the width of the river.

$$\begin{aligned} BC &= CA \times \tan A. \\ &= 235 \times \tan 75^{\circ} 30' \\ &= \frac{235}{\tan 15^{\circ} 30'}. \\ &= 847 \text{ ft.} \end{aligned}$$

LOGARITHMS.

Between the scale of sines and the scale of tangents is a scale of equal parts by which the logarithm of a number may be found.

Example 1. Find $\log 50$.

With the slide in its usual position having the scale of equal parts underneath, set the left index of the slide to 5 on D. On the scale of equal parts opposite the right index on the under side of the rule, read $\log 5 = 699$. Placing the decimal point and prefixing the characteristic, as usual in working with logarithms, $\log 50 = 1.699$.

(The characteristic is found by taking one less than the number of figures at the left of the decimal point.)

Example 2. Find $(2.36)^5 = x$.

$$\begin{aligned}\log x &= 5 \times \log 2.36. \\ &= 5 \times .373. \\ &= 1.865. \\ x &= 73.2.\end{aligned}$$

Example 3. Find $\sqrt[5]{187} = x$.

$$\begin{aligned}\log 187 &= 2.272. \\ \log \sqrt[5]{187} &= 1/5 \text{ of } 2.272 = .454. \\ x &= 2.84.\end{aligned}$$

EXERCISE.

Find the logarithms of

1. 1.24.
 2. 54.5.
 3. .312.
 4. .067.
 5. 735.
- Find the value of the following to three significant figures:
6. $(3.2)^5$.
 7. $(425)^4$.
 8. $\sqrt[3]{3.46}$.
 9. $\sqrt[5]{286}$.
 10. $\sqrt[7]{1430}$.

CHAPTER III.

TYPICAL PROBLEMS FROM VARIOUS OCCUPATIONS.

Secretarial Work. A secretary in checking a traveling man's expense account for one week found items as follows:

Railroad fares.....	\$27.50
Hotel bills.....	56.00
Total.....	83.50

Find what per cent of the total expense was used in hotel bills.

Solution: $56 \div 83.50 = 67$ per cent.

ESTIMATING PLASTERING.

As a detail in estimating the cost of plastering a house, find the number of square yards in the walls and ceiling of a room 24 ft. long, 16 ft. wide, and 9 ft. high without any allowance for openings.

Ceiling, 24×16	= 384 by the slide rule.
Walls, $2 \times (24 \times 16) \times 9$	= 720
Total.....	1104

EXCAVATING.

What will be the cost of excavating rock for a cellar measuring 43 ft $\times$ 23 ft. to an average depth of 6.5 ft. at \$2.50 per cubic yard?

$$x = \frac{43 \times 28 \times 6.5 \times 2.5}{27}.$$

$$= \$725. \text{ Correct to the nearest dollar.}$$

BUSINESS

PER CENT OF PROFIT.

A merchant purchased a bill of goods for \$318 and sold the same for \$360. Find the per cent of profit reckoned,

- a. On the cost.
- b. On the selling price.

Solution: Profit = \$360 — \$318 = \$42.

Per cent of profit reckoned on the cost = $\frac{42}{318} = 13.2$ per cent.

Per cent of profit reckoned on the selling price = $\frac{42}{360} = 11.7$ per cent.

DISCOUNT.

Goods marked \$7.25 are sold at a discount of 35½ per cent. Find the net price.

Solution: The net price is 100 per cent—35½ per cent or 64½ per cent.
 $.645 \times 7.25 = 4.68.$

COMPOUND INTEREST.

How many years will it take a sum of money to double itself if deposited in a savings bank paying 4 per cent interest, compounded semi-annually.

Using the formula $A = P(1 + r)^n$, where A is the amount, P the principal, r the interest of \$1. for 6 months, and n the number of half years, if we take \$1. as P , we have:

$$2 = (1 + .02)^n.$$

$$\text{and } n = \frac{\log 2}{\log 1.02}.$$

$$= \frac{.301}{.0086}.$$

$$= 35 \text{ half years.}$$

$$\text{or } 17\frac{1}{2} \text{ years.}$$

NOTE.—It is advisable to use a 20-inch rule for this problem. On the 10-inch rule the result can only be found very roughly.

PHYSICS.

In a photometer a 16 c. p. lamp is used as a standard. The following distance readings are obtained in testing a nitrogen filled lamp.

D_s	D_x	By experiment
317 mm.	683 mm.	1
304 mm.	696 mm.	2
322 mm.	678 mm.	3
248 mm.	702 mm.	4

Using the following equation calculate the observed candle power of the unknown lamp.

SOLUTION.

$$\begin{array}{rcl} D_s^3 & \text{c. p. of standard} & \\ \hline D_x^3 & \text{c. p. of unknown} & \\ (317)^2 & 16 & \\ \hline (683)^2 & x & \end{array}$$

To 317 on scale *D* set 683 on *C*.

Above 16 on *B* find *x* on *A*.

$$x = 34.5$$

The operation of transferring from scales *C* and *D* to *A* and *B* squares the

fraction $\frac{317}{683}$.

The first experiment gives	$x = 34.5$
The second "	$x = 31.0$
" third "	$x = 36.0$
" fourth "	$x = 28.8$

$$4 \overline{)130.5}$$

$$\text{The result} \dots\dots\dots = 32.6$$

CHEMISTRY.

By weight 80 parts of sodium hydroxide combines with 98 parts of sulphuric acid. How many grams of sodium hydroxide will neutralize 50 grams of sulphuric acid?

$$\text{Solution: } 80 : 98 = 50 : x.$$

To 98 on *D* set 80 on *C*.

Under 50 on *C* find 61.25 on *D*.

MACHINE SHOP.

SPEEDS OF PULLEYS.

The diameter of the driving pulley is 9 inches and its speed is 1,300 R. P. M. If the diameter of the driving pulley is 7 inches, what is its speed?

Solution: The diameter of the driving pulley, multiplied by its speed, is equal to the diameter of the driven pulley, multiplied by its speed.

$$\begin{array}{rcl} 7 \text{ S} & = & 9 \times 1300 \\ \text{S} & = & 9 \times 1300 \end{array}$$

$$\underline{\hspace{1cm}} \\ 7$$

By using the method of Page 10:

$$\text{S} = 1670 \text{ correct to three significant figures.}$$

CUTTING SPEED.

A certain grindstone will stand a surface or rim speed of 800 ft. per min. At how many R. P. M. can it run if its diameter is 4 ft. 9 in.?

Solution: The cutting speed is equal to the circumference of the work in feet multiplied by the number of revolutions per minute.

or $C = \frac{\pi d \times \text{R. P. M.}}{12}$ where d is expressed in inches.

$$\begin{aligned} \text{Hence R. P. M.} &= \frac{12 C}{\pi d} \\ &= \frac{12 \times 800}{3.1416 \times 4.75} \\ &= 644. \end{aligned}$$

GEARING.

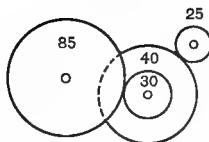


Fig. 38.

The gear with 85 teeth (Fig. 38) revolves 50 times per minute. Find the speed of the gear with 25 teeth.

Solution: The continued product of the R. P. M. of the first driver and the number of teeth in every driving gear is equal to the continued product of the R. P. M. of last driven gear and the number of teeth in every driven gear.

$$\begin{aligned} \text{Hence, } 50 \times 85 \times 40 &= 30 \times 25 \times S \\ S &= \frac{50 \times 85 \times 40}{30 \times 25} \\ S &= 227. \end{aligned}$$

LENGTH OF PATTERN.

If window weights are $1\frac{1}{2}$ inches in diameter, how long must we make the pattern for 8 lb. weights (1 cu. in. of cast iron weighs .26 lb.)?

Solution: The number of lb. in the window weight is equal to the volume of the cylindrical weight $\times .26$ lb.

$$\begin{aligned} 8 &= \frac{\pi \times (1.5)^2 \times L \times .26}{4 \times 8} \\ \text{and } L &= \frac{\pi \times (1.5)^2 \times .26}{4 \times 8} \\ &= 17.4 \text{ inches, or } 17 \text{ and } 7/16 \text{ inches.} \end{aligned}$$

FORGE SHOP.

Find the length of stock for making a welded ring washer of four inches *inside* diameter. Stock to be used $1\frac{3}{8}'' \times \frac{1}{4}''$ flat.

NOTE.—The diameter to be used is always the mean diameter, *i. e.*, measured from center line to center line of the stock. Thus, $4''$ (inside diameter) $+ 11/16'' + 11/16'' = 5\frac{3}{8}''$, mean or working diameter. The mean circumference, and therefore, the required length of stock is thus 5.38×3.14 .

Since this ring washer is to be welded, an allowance of $\frac{1}{4}$ inch must be made for the weld equal to the thickness of stock. Set right hand index to 314 on *D*. Under 5.38 on *C* find 16.90 on *D*. Allowing $\frac{1}{4}$ inch for welding, we have $16.90 + .25 = 17.15$ inches.

MIXING PROBLEMS.

If bell metal is made of 25 parts of copper to 11 parts of tin in weight, find the weight of each metal in a bell weighing 1054 lbs.

Solution: The copper weighs $\frac{25}{36}$ of 1054 = 734 lbs.

The tin weighs $\frac{11}{36}$ of 1054 = 321 lbs.

SURVEYING.

The slide rule is used in surveying to check gross errors in computation; to reduce stadia readings, and to solve triangles where results to three significant figures are sufficiently accurate, using the 10'' rule, or for four significant figures, using the 20'' rule.

See page 62 for the solution of triangles by the slide rule.

Example 1. Find the latitude and departure of a course whose length is 525 ft. and bearing N 65° 30' E.

$$\begin{aligned}\text{Lat.} &= \text{Length of course} \times \cos \text{ of bearing.} \\ &= 525 \times \cos 65^\circ 30'. \\ &= 525 \times \sin 24^\circ 30'. \\ &= 218.\end{aligned}$$

To the mark in the groove at the right of the rule set 24° 30' on scale *S*.

Under 525 on *A*, find 218 on *B*.

The decimal point may be placed by inspection, since the sine and cosine are always less than one.

$$\begin{aligned}\text{Dep.} &= \text{Length of course} \times \sin \text{ of bearing.} \\ &= 525 \times \sin 65^\circ 30'. \\ &= 478.\end{aligned}$$

We now give, by permission of *Engineering News*, a number of settings for complicated formulas, and also a very useful and complete table of Equivalents or Gauge Points, specially adapted to Slide Rule practice.

These will probably enable any one to work out similar Equivalents for any special class of calculations.

FORMULAS.

$$\frac{a^2 \times b}{c} = X.$$

SETTINGS.

A		Find X
B	Set c	Over b
C		
D	To a	

To *a* on *D*, set *c* on *B*. Transferring from *C* and *D* to *A* and *B* has the effect of squaring *a*.

Over *b* on *B*, find *x* on *A*.

$$\sqrt{\frac{a \times b}{c}} = X$$

Inverted

A	To b	
C		Under c
B	Set a	
D		Find X

Multiplication becomes division with the slide inverted as in the diagram.

$$\sqrt{\frac{a \times b}{c}} \text{ is solved like } \sqrt{\frac{b}{a}} \times c.$$

Transferring from scales *A* and *B* to *C* and *D*, obtains the square root of $\frac{b}{a}$.

Inverting the slide makes it possible to solve problems of this type with one setting of the slide where two would be required with the slide in the usual position.

$$\frac{a \sqrt{b}}{c} = X$$

Inverted

A	Under b	
C	set a	Under c
B		
D		Find X

$$c \sqrt{\frac{a \times b}{d}} = X.$$

A	Under a			
B	Set d	R to b	1 to R	-
C				Under c
D				Find X



Inverted	A	Under b		
	C		c to R	Under 1
	B	Set a	R to d	
	D			Find X

$$\frac{\sqrt{a^3}}{c} = \frac{\sqrt{a \times a}}{c}$$

Inverted	A		
	C		Under c
	B	Set a	
	D	To a	Find X

EXAMPLES

Let $a = 4$,

and $c = 3$.

Setting 4 on B to 4 on D gives $4^3 = 64$ on A over 1 on B .

Dropping down to D , opposite 64, gives $\sqrt{4^3}$, or 8 on D .

Moving the runner to 3 on C gives $\frac{\sqrt{4^3}}{3}$, or 2.67 on D .

$$c\sqrt{b^3} = X$$

A			
B		R to b	
C	Set 1	1 to R	Under c
D	To b		Find X

$$\left\{ \frac{\sqrt{a \times b}}{c} \right\}^2 = X$$

Inverted	A	Under a	Find X
	C	Set b	Over c
	B		
	D		

$$\sqrt{\frac{a^2 \times b^2}{c}} = X$$

Inverted	A		
	C	Set b	
	B		Under c
	D	To a	Find X



To find the Geometric Mean, or Mean Proportional between two numbers,
or $a : x :: x : b$

A		
B	Set less No. a	Below b
C		
D	To less No. a	Find $X = G. M.$

NOTE.—In operations involving square root care should be taken to move the decimal point an even number of places and to use the proper right or left half of A or B.

To reduce fractions to decimals:

C	Set numerator	Find equivalent decimal
D	To denominator	Above 1

To reduce decimals to fractions:

C	Set decimal	Find equivalent numerators.
D	To 1	Find equivalent denominators.

EQUIVALENTS FOR SCALES C AND D. GEOMETRICAL

Set 226 = Diameters of circles.

To 710 = Circumference of circles.

79 = Diameter of circle.

70 = Side of equal square.

99 = Diameter of circle.

70 = Side of inscribed square.

39 = Circumference of circle.

11 = Side of equal square.

40 = Circumference of circle.

9 = Side of inscribed square.

70 = Side of square.

99 = Diagonal of square.

205 = Area of square whose side = 1.

161 = Area of circle whose diameter = 1.

322 = Area of circle.

205 = Area of inscribed square.

ARITHMETICAL.

100 = Links.

66 = Feet.

12 = Links.

95 = Inches.

101 = Square links.

44 = Square feet.

6 = U. S. Gallons.

5 = Imperial gallons.

1 = U. S. gallons.

231 = Cubic inches.

800 = U. S. gallons.

107 = Cubic feet.

22 = Imperial gallons.

6100 = Cubic inches.

430 = Imperial gallons.

69 = Cubic feet.

METRIC SYSTEM.

26 = Inches.

66 = Centimeters.

82 = Yards.

75 = Meters.

4300 = Links.

865 = Meters.

82 = Feet.

25 = Meters.

87 = Miles.

140 = Kilometers.

43 = Chains.

865 = Meters.

31 = Square inches.

200 = Square Centimeters.

140 = Square feet.

13 = Square meters.

61 = Square yards.

51 = Square meters.

42 = Acres.

17 = Hectares.

22 = Square miles.

57 = Square kilometers.

5 = Cubic inches.

82 = Cubic centimeters.

600 = Cubic feet.

17 = Cubic meters.

85 = Cubic yards.

65 = Cubic meters.

6 = Cubic feet.

170 = Liters.

14 = U. S. gallons.

53 = Liters.

46 = Imperial gallons.

209 = Liters.

108 = Grains.

7 = Grams.

75 = Pounds.

34 = Kilograms.

6 = Ounces.

170 = Grams.

63 = Hundredweights.

3200 = Kilograms.

63 = English tons.

64 = Metric tons.

PRESSURES.

640 = Pounds per square inch.

45 = Kilogs per square centimeter.

51 = Pounds per square foot.

249 = Kilogs per square meter.

59 = Pounds per square yard.

32 = Kilogs per square meter.

57 = Inches of mercury.

28 = Pounds per square inch.

82 = Inches of mercury.

5800 = Pounds per square foot.

720 = Inches of water.

26 = Pounds per square inch.

74 = Inches of water.

385 = Pounds per square foot.

60 = Feet of water.

26 = Pounds per square inch.

5 = Feet of water.

312 = Pounds per square foot.

15 = Inches of mercury.

17 = Feet of water.

99 = Atmospheres.

2960 = Inches of mercury.

34 = Atmospheres.

500 = Pounds per square inch.

34 = Atmospheres.

7200 = Pounds per square foot.

30 = Atmospheres.

31 = Kilogs per square centimeter.

23 = Atmospheres.

780 = Feet of water.

3 = Atmospheres.

31 = Meters of water.

29 = Pounds per square inch.

67 = Feet of water.

1 = Kilogs per square centimeter.

10 = Meters of water.

COMBINATIONS.

43 = Pounds per foot.

64 = Kilogs per meter.

127 = Pounds per yard.

63 = Kilogs per meter.

46 = Pounds per square yard.

25 = Kilogs per square meter.

49	= Pounds per cubic foot.
785	= Kilogs per cubic meter.
27	= Pounds per cubic yard.
16	= Kilogs per cubic meter.
89	= Cubic feet per minute.
42	= Liters per second.
700	= Imperial gallons per minute.
53	= Liters per second.
840	= U. S. gallons per minute.
53	= Liters per second.
38	= Weight of fresh water.
39	= Weight of sea water.
5	= Cubic feet of water.
312	= Weight in pounds.
1	= Imperial gallons of water.
10	= Weight in pounds.
3	= U. S. gallons of water.
25	= Weight in pounds.
50	= Pounds per U. S. gallon.
6	= Kilogs per liter.
10	= Pounds per Imperial gallon.
1	= Kilogs per liter.
30	= Pounds per U. S. gallon.
25	= Pounds per Imperial gallon.
3	= Cubic feet of water.
85	= Weight in kilogs.
46	= Imperial gallons of water.
209	= Weight in kilogs.
14	= U. S. gallons of water.
53	= Weight in kilogs.

44 = Feet per second.

30 = Miles per hour.

88 = Yards per minute.

3 = Miles per hour.

41 = Feet per second.

750 = Meters per minute

82 = Feet per minute.

25 = Meters per minute.

340 = Footpounds.

47 = Kilogrammeters.

72 = British horse power.

73 = French horse power.

3700 = One cubic foot of water per minute under one foot of head.

7 = British horse power.

75 = One liter of water per second under one meter of head.

1 = French horse power.

In no case does the departure, in these equivalents, from the exact ratio attain one per thousand.

EXAMPLES.

What is the pressure in pounds per square inch equivalent to a head of 34 feet of water?

C		Set 60		Under 34
D		To 26		Find 14.75 pounds—Answer.

What head of water, in feet, is equivalent to a pressure of 18 pounds per square inch?

C		Set 26		Under 18
D		To 60		Find 41.5 feet—Answer.

How many horse power will 50 cubic feet of water per minute give under a head of 400 feet?

C		Set 3700		Runner to 400		1 to R		Under 50
D		To 7						Find 37.8 H. P.—Answer.

METHODS OF WORKING OUT MECHANICAL AND OTHER FORMULAS.

1. Diameters and Areas of Circles. $A = .7854 D^2$.

A	To 205		A	To 11	
B	Set 161	Find Areas.	B	Set 6	Find Areas in square feet.
C			C		
D		Above Diameters	D		Above Diameter in inches.

2. To Calculate Selling Prices of Goods, with percentage of profit on Cost Price.

C	Set 100	Below cost price.
D	To 100 plus percentage of profit	Find selling price.

3. To Calculate Selling Prices of Goods, with percentages of profit on Selling Price.

C	Set 100 less percentage of profit	Below cost price
D	To 100	Find selling price

Example: If goods cost 45 cents a yard, at what price must they be sold to realize 15 per cent profit on the selling price?

C	Set 85 (=100—15)	Below 45
D	To 100	Find 53—Answer

4. To find the Area of a Ring. $A = \frac{(D + d) \times (D - d)}{1.2732}$

C	Set sum of the two diameters	Find area.
D	To 1.273	Above difference of the two diameters

5. Compound Interest.

Set the left index of C, to 100 plus the rate of interest, on D, then take the corresponding number on the scale of Equal Parts, and multiply it by the number of years. Set this product on the scale of E. P. to the index on the under side of the Rule, then on D will be found the amount of any coinciding sum on C for the given years at the given rate.

Example: Find the amount of \$150 at 5 per cent at the end of 10 years.

C	Set 100	E. P. = $21.2 \times 10 = 212$	212 to 1	C	Below 150
D	To 105	Under side of Rule and	Slide	D	Find \$244.35—Ans.

We thus obtain on D, below 1 on C, a gauge-point for 10 years at 5 per cent. and can obtain in like manner similar ones for any other number of years and rate of interest.

6. Levers.

C	Set distance from fulcrum to power or weight applied	Find power or weight transmitted.
D	To distance from fulcrum to power or weight transmitted	Above power or weight applied.

7. Diameter of Pulleys or Teeth of Wheels

Inv. C	Set diameter or teeth of driving	Diameter or teeth of driven
D	To revolutions of driving	Revolutions of driven
C	Set diameter or teeth of driving	Revolutions of driven
or, D	To diameter or teeth of driven	Revolutions of driving

8. Diameter of two Wheels to work at given Velocities.

C	Set distance between their centers	Find diameter.
D	To half sum of their revolutions	Above revolutions of each

9. Strength of Teeth of Wheels. $P = \frac{\sqrt{H}}{0.6V}$

A	To H. P. to be transmitted			
B				
C	Set gauge point 0.6	R to 1	Velocity in ft. per second to R	Under 1
D				Pitch in inches

10. Diameter and Pitch of Wheels. $N = \frac{D \times \pi}{P}$

C	Set 226	R to 1	Pitch to R	Under diameter of pitch circle
D	To 710			Find number of teeth

11. Strength of Wrought Iron Shafting.

$D = \sqrt[3]{\frac{83 H}{N}}$ for crank shafts and prime movers.

$D = \sqrt[3]{\frac{65 H}{N}}$ for ordinary shafting

A	To 83 or 65		
B	Set revolutions per min.	R to Ind. H. P.	Number coinciding with R = diameter
C			Under 1
D			Same coinciding number = diameter

NOTE.—In this, as in other cases, the coefficients (83 and 65) may be altered to suit individual opinions, without in any way altering the methods of solution.

12. To find the Change Wheel in a Screw-Cutting Lathe.

$$N = T \frac{S \times W}{M \times P} \quad \text{where} \quad \begin{cases} N = \text{Number of threads per inch to be cut.} \\ T = \text{“ “ “ on traverse screw.} \\ M = \text{“ “ “ teeth in wheel on mandril.} \\ W = \text{“ “ “ stud wheel (gearing in M).} \\ P = \text{“ “ “ stud pinion (gearing in S).} \\ S = \text{“ “ “ wheel on traverse screw.} \end{cases}$$

$$W = N \frac{M \times P}{T = S}$$

C	Set T	R to P	S to R	Under M
D	To N			Find No. of teeth in W or stud wheel

13. Rules for Good Leather Belting. $W = \frac{600 \text{ or } 375 \text{ H. P.}}{V \text{ ft. per min.}}$

C	Set 600	Find width in inches	for Single Belts
D	To velocity in feet per min.	Above actual H. P.	
C	Set 375	Find width in inches	for Double Belts
D	To velocity in feet per min.	Above actual H. P.	

14. Best Manila Rope Driving.

A	To velocity in feet per min.	Find Actual Horse Power.
B	Set 307	
C		Above diameter in inches
D		

A	To 4	Find Strength in Tons.
B		
C	Set 1	Above diameter in inches
D		
A	To 107	Find Working Tension in Pounds.
B		
C	Set 1	Above diameter in inches
D		
A	To 0.28	Find Weight per Foot in Pounds.
B		
C	Set 1	Above diameter in inches
D		

15. Weight of Iron Bars in Pounds per Foot Length.

A	To 1	Weight of Square Bars.
B	Set 3	
C		Above width of side in inches
D		

A	To 55	Weight of Round Bars.
B	Set 21	
C		Above diameter in inches
D		

C	Set 0.3	Below thickness in inches
D	Breadth in inches	Weight of Flat Bars.

16. Weight of Iron Plates in Pounds per Square Foot.

C	Set 32	Below thickness in thirty-seconds of an inch
D	To 40	Find weight in pounds per square foot.

17. Weights of other Metals.

C	Set 1	Below G. P. for other metals
D	To weight in iron	Find weight in other metals.

Gauge-points of other metals, and weight per cubic foot.

	W. 1.	C. 1.	Cast Steel.	Steel Plates.	Copper.	Brass.	Lead.	Zinc
G. P.....	1	.93	1.02	1.04	1.15	1.09	1.47	.92
Weight....	480	450	490	500	550	525	710	440 pounds.

Example: What is the weight of a bar of copper, 1 foot long, 3 inches broad and 2 inches thick?

C	Set 0.3	R to 2 inches thick	1 to R	Below G. P. 1.15
D	To 3 inches broad			Find 23 pounds—Ans.

18. Weight of Cast Iron Pipes.

C	Set .4075	Below Difference of inside and outside diameters in inches
D	To Sum of inside and outside diameters in inches	Find weight in pounds per lineal foot.

G. P. for other metals.....	Brass.	Copper.	Lead.	W. Iron
	.355	.333	.259	.38

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19. Safe Load on Chains.

A		Safe load in tons.
B	Set 36	Above 1
C		
D	To diameter in sixteenths of an inch	

20. Gravity.

C	Set 1	Below 32.2
D	To seconds	Velocity in feet per second.

A	Space fallen through in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second.

A		Space fallen through in ft.
B		Above 16.1
C	Set 1	
D	To seconds	

21. Oscillations of Pendulums.

A		
B	Set length pendulum in in.	
C		Below 1
D	To 375	Number oscillations per minute.

22. Comparison of Thermometers.

C	Set 5	Degrees Centigrade.
D	To 9	Degrees + 32 = Fahrenheit.

C	Set 4	Degrees Reaumur
D	To 9	Degrees + 32 = Fahrenheit.

C	Set 4	Degrees Reaumur
D	To 5	Degrees Centigrade.

23. Force of Wind.

A		
B	Set 10	Find pressure in pounds per square foot
C		
D	To 66	Velocity in <i>Feet per second</i> .

A		
B	Set 10	Find pressure in pounds per square foot
C		
D	To 45	Velocity in <i>Miles per hour</i> .

24. Discharge from Pumps.

A		Gallons delivered per stroke.
B	Set 294	Stroke in inches
C		
D	To diameter in inches	

25. Diameter of Single-acting Pumps.

A	To 294			
B	Set length of stroke in inches.	R to gallons to be delivered per min.	No. strokes per min. to R	
C				Below 1
D				Diam. pump in inches.

26. Horse Power required for Pumps.

C	Set G. P.	Height in feet to which the water is to be raised.
D	To cubic feet or gallons to be raised per minute	Horse power required.

Gauge Points with different percentages of allowance.

Per Cent.....	None	10	20	30	40	50	60	70	80
For Gallons Imp...	3300	3000	2750	2540	2360	2200	2060	1940	1835
" C. Feet.....	528	480	440	406	377	352	330	311	294
" U. S. Gallons..	3960	3600	3300	3050	2830	2640	2470	2330	2200

27. Theoretic Velocity of Water for any Head.

A	Head in feet	
B		
C	Set 1	Under 8
D		Velocity in feet per second.

28. Theoretical Discharge from an Orifice 1 inch Square.

A			If the hole is round and one inch dia. the G. P. is 2.62.
B	Set 1	Under head in feet	
C			
D	To G. P. 3.34	Discharge in cubic feet per minute.	

29. Real Discharge from Orifice in a Tank 1 inch Square.

A			If the hole is round and 1 inch diam., the G. P. is 1.65.
B	Set 1	Under head in feet.	
C			
D	To 2.1 G. P.	Discharge in cubic feet per minute with coefficient .63.	

Gauge Points for other coefficients.

Coefficient.....	.60	.66	.69	.72	.75	.78	.81	.84	.87	.90	.93	.96
G. P. Square.....	2.	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3.	3.1	3.2
" Round.....	1.57	1.73	1.80	1.88	1.96	2.04	2.12	2.20	2.28	2.36	2.44	2.52

30. Discharge from Pipes when real velocity is known.

Inverted	A		Discharge in cubic feet per min.
	C		Above 1.75
	B	Velocity in feet per second	
	D	Diameter in inches	

31. Delivery of Water from Pipes.

$$W = 4.71 \sqrt{\frac{D^5 H}{L}}$$

Eytelwein's Rule.

C	Set 1	$EP \times 5 = x$	x to I
D	To diameter in inches	Under side of Rule.	(Con. to next line.)

A				To Z	
B				Set 1	
C	R to 1	Length in feet to R	Under head in feet		Under 4.71
D			Find Z		Cu.ft.per min.

This is worked out similarly to Formula 5, which is explained in full.

32. Gauging Water with a Weir.

Inverted	A		
	C		
	B	Depth in inches	Under 4.3
	D	Depth in inches	Discharge in cubic feet per minute from each foot width of sill.

33. Discharge of a Turbine.

$$\frac{\sqrt{H \times V}}{0.3} = D$$

Inverted	A		
	C		Under 0.3
	B	Head in feet	
	D	Square inches water vented	Cubic feet discharged per minute,

34. Revolutions of a Turbine.

A	To head in feet			
B				
C	Set diameter in inches	R to 1840	1 to R	Under rate of peripheral velocity
D				Find revolutions per min.

35. Horse Power of a Turbine.

C	Set 530	R to discharge per c. ft. per min.	1 to R	Percentage useful effect
D	Head in ft.			Horse power.

A	Under head in ft.				
B					
C	Set 1	R to head in ft.	158 to R	R to vent in sq. in.	to R Under useful effect
D					Horse power

36. Horse Power of a Steam Engine,

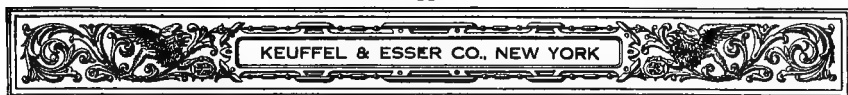
C	Set 21,000	R to diam.	1 to R	R to stroke in ft.	1 to R	R to rev. per min.	1 to R	Mean pressure per sq. inch
D	To diam. in inches							Horse power.



or,	A					H. P.
	B	Set 21,000	R to stroke in ft.	1 to R	R to revolutions	1 to R
	C					Mean pressure
	D	To diam. in inches				

37. Dynamometer to Estimate the H. P. indicated by

C	Set 5252	R to length of lever in feet from center of shaft	1 to R	Under rev. of shaft per min.
D	weight applied at end of lever in pounds, including weight of scale.			actual horse power.



SUPPLEMENT

THE SLIDE RULE AND PLANE TRIGONOMETRY.

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The Slide Rule is commonly used by engineers to solve the numerical problem

$$a : b = c : x$$

but it does not seem to be generally known that practically every problem of plane trigonometry, including the more difficult solutions of oblique triangles, can be made *with one setting* of the ordinary Slide Rule.

It is proposed here to take up the different cases usually treated in plane trigonometry, giving an illustration of each.

1. Given degrees, to find the corresponding radian measure of an angle, or the reverse.

A	on π	Read any radians
B	Set 180	Read any degrees
C		
D		



2.	A	To 10	Under 7
arc sec ($\frac{1}{7}$)	S	Set 1 (sin 90°)	Read (90° —answer) $45^\circ.5$
	T		
	D		

PLANE RIGHT TRIANGLES.

3. Given two legs. Using the usual notation A, B, C , for angles; a, b, c , for sides opposite

$$\text{Let } a = 0.0227, b = 0.084, C = 90^\circ$$

A		
S		
T	Set 1 (tan 45°)	Read A
D	Over 84	Over 227

$$A = 15^\circ 7'$$

$$B = 90^\circ - (15^\circ 7') = 74^\circ 53'. \text{ To find } c$$

A	Under 227	Read answer $c = 0.087$
S	Set $15^\circ 7'$	Over 1 (sin 90°)
T		
D		

If either angle is less than 6° the sine may be used for the tangent as follows:

4. Let $a = 64.4$ $b = 5.99$.

A	To 64.4	Under 5.99
S	Set 1 (sin 90°)	Read answer $B = 5^\circ 19'$
T		
D		

$$B = 5^\circ 19' \quad A = 90^\circ - (5^\circ 19') = 84^\circ 41'$$

5. Given a leg and the hypotenuse.

$$\text{Let } c = 7.46 \quad b = 0.647$$

A	Under 746	Under 647	Read a
S	Set 1 ($\sin 90^\circ$)	Read B	Over $85^\circ 1'$
T			
D			

$$B = 4^\circ 59'$$

$$A = 90^\circ - (4^\circ 59') = 85^\circ 1'$$

$$a = 7.43$$

6. If a only is required in the last example it may be better to use the formula $a = \sqrt{(c+b)(c-b)}$

A	To 8.107	
B	Set 1	Under difference 6.813
C		
D		Read answer $a = 7.43$

7. When b is nearly equal to c , we can find A more accurately by the formula

$$\sin^2 \frac{A}{2} = \frac{c-b}{2c}$$

$$\text{Let } b = 7.4169 \quad c = 7.4451$$

A	To $c-b=0.0282$	Index to 2.295
B	Set $2c=14.8902$	Read answer $\frac{A}{2} = 2^\circ 29'$
C		
D		Read 2.295

8. Given a leg and an acute angle.

$$\text{Let } b = 0.084 \quad A = 15^\circ 9'$$

$$\text{whence } B = 74^\circ 51'$$

A	To 0.084	Read $c = 0.087$	Read $a = 0.0227$
S	Set $74^{\circ} 51'$	Over 1 ($\sin 90^{\circ}$)	Over $15^{\circ} 9'$
T			
D			

If only a is desired we read more accurately

A		
S		
T	Set 1 ($\tan 45^{\circ}$)	Under $15^{\circ} 9'$
D	Over 0.084	Read $a = 0.0227$

9. Given the hypotenuse and an acute angle.

Let $c = 426$ $A = 34^{\circ} 15'$

whence $B = 55^{\circ} 45'$

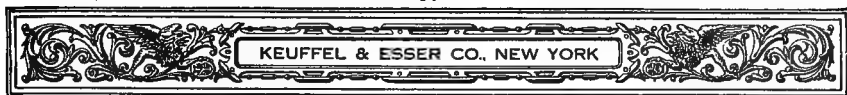
A	On 426	Read $a = 240$	Read $b = 353$
S	Set 1 ($\sin 90^{\circ}$)	Over $34^{\circ} 15'$	Over $55^{\circ} 45'$
T			
D			

10. Latitude and departure of plane surveying is a special case of right triangles:

A	On "distance"	Read "departure"	Read "latitude"
S	Set 1 ($\sin 90^{\circ}$)	Over "course"	Over (90° —course)
T			
D			

11. Projecting a line (or stress) on another line.

A	On length	Read projection
S	Set 1 ($\sin 90^{\circ}$)	Over (90° —angle)
T		
D		



12. Let a force of 5 lbs. act at an angle of 59° to another line. Determine its resolute.

A	On 5	Read answer = 2.58
S	Set 1 ($\sin 90^\circ$)	Over 31°
T		
D		

PLANE OBLIQUE TRIANGLES.

13. Given a side and two adjacent angles.

Using the formula $a : \sin A = b : \sin B = c : \sin C$.

Let $a = 25$, $B = 50^\circ$, $C = 60^\circ$; whence $A = 70^\circ$

A	On 25	Read $b = 20.6$	Read $c = 23.2$
S	Set 70°	Over 50°	Over 60°
T			
D			

14. Given two sides and an angle opposite one of them (case of double solution). The minutes of arc cannot be read correctly to the unit with the slide rule.

Let $A = 48^\circ 34'$, $a = 46.24$, $b = 60.02$

A	To 46.24	Under 60.02	Read $c = 50.36$	Read $c_2 = 29.08$
S	Set $48^\circ 34'$	Read B $B_2 = 180^\circ - (76^\circ 42')$	Over $54^\circ 44'$ $B_1 = 76^\circ 42', C_1 = 54^\circ 44'$	Over $28^\circ 8'$ $B_2 = 103^\circ 18'$ $C_2 = 28^\circ 8'$
T				
D				

15. Sometimes in the double solution case we have only one solution.

Let $A = 32^\circ 43'$, $a = 988$, $b = 672$.

Here A is acute and $a > b$, hence we have only one solution as follows:

A	On 988	Under 672	Read $c = 1484$
S	Set $32^\circ 43'$	Read B	Over $54^\circ 17'$
T			
D			

$$B = 21^\circ 34'$$

$$C = 180^\circ - (A + B) = 180^\circ - (54^\circ 17')$$

16. Given two sides and the included angle.

Use the formula

$$\frac{\tan \frac{B-C}{2}}{b-c} = \frac{\cot \frac{A}{2}}{b+c}$$

The fact that the tangent scale runs only to 45° makes two cases.

Case I. $\frac{A}{2}$ greater than 45° .

Let $b=6.24$, $c=2.35$, $A=110^\circ 32'$, whence $\frac{A}{2} = 55^\circ 16'$. $\frac{B+C}{2} = 90 - \frac{A}{2} = 34^\circ 44'$, $b+c = 8.59$, $b-c = 3.89$.

A		
S		
T	Set $34^\circ 44'$	Read $\frac{B-C}{2} = 17^\circ 26'$
D	On 8.59	Over 3.89

Adding and subtracting $\frac{B+C}{2}$, $\frac{B-C}{2}$

$$B = 52^\circ 10'$$

$$C = 17^\circ 18'$$

Case II. $\frac{A}{2}$ less than 45° .

A			
S			
T	Set $1(\tan 45^\circ)$	Runner to $\frac{A}{2}$	Read $\frac{B-C}{2}$
D	On $b+c$	Index to runner	Over $b-c$

Let $b=226.2$, $c=138.7$, $A=59^\circ 13'$, whence $b+c=364.9$, $b-c=87.5$,

$$\frac{A}{2} = 29^\circ 36', \quad \frac{B+C}{2} = 60^\circ 24'$$

A			
S			
T	Set $1(\tan 45^\circ)$	Runner to $29^\circ 36'$	Read $\frac{B-C}{2} = 22^\circ 54'$
D	On 364.9	Index to runner	Over 87.5

Adding and subtracting $\frac{B+C}{2}$, $\frac{B-C}{2}$

$$B = 83^\circ 18'$$

$$C = 37^\circ 30'$$

17. Given the three sides.

The common solution requires first the finding of the radius of the inscribed circle, by the formula

$$r^2 = \frac{(s-a)(s-b)(s-c)}{s} \quad s = \frac{a+b+c}{2}$$

and then the angles, by the formulas

$$\tan \frac{A}{2} = \frac{r}{s-a}, \quad \tan \frac{B}{2} = \frac{r}{s-b}, \quad \text{etc.}$$

This is the only problem of plane triangles which requires several settings of the slide rule for its solution. But after r is found *one setting* gives any angle, as follows:

Let $a=260$, $b=280$, $c=300$, whence $s=420$, $s-a=160$, $s-b=140$, $s-c=120$.

A	On 160	1 to runner	
B	Set 420	Runner to 140	Under 120
C			
D			Read $r = 7.99$

Invert the slide.

A		
S		
T	Set 1	Read $\frac{A}{2} = 26^\circ 34'$
D	On 160	Over $r (= 7.99)$

$$A = 53^\circ 08'$$

This completes the solution of all cases of plane triangles.

ANSWERS.

CHAPTER I.

(Answers given with the problems are not given below.)

- | | |
|-----------|---------|
| 16. 300. | 20. .03 |
| 17. 3000. | 21. 3. |
| 18. 3000. | |
| 19. .3 | |
| 27. | |

	31	32	33	34	35
21	651	672	693	714	735
22	682	704	726	748	770
23	713	736	759	782	805
24	744	768	792	816	840
25	775	800	825	850	875
26	806	832	858	884	910
27	837	864	891	918	945
28	868	896	924	952	980
29	899	928	957	986	1015
30	930	960	990	1020	1050

- | | |
|---------------|------------------------------|
| 30. 49% | 73. 21.6 |
| 31. 33% | 74. 1.25 |
| 32. 91% | 75. 74.8 |
| 33. 58% | 76. 76200. |
| 34. 16% | 77. 1170. |
| 35. 21% | 78. .436 |
| 39. 2.24 | 79. .0039 |
| 40. 2.34 | 80. .0000325 |
| 41. 1.33 | 81. .000595 |
| 42. 1.32 | 82. 5020000. |
| 43. 3.18 | 91. 1.19 |
| 44. 67.3 | 92. 3.76 |
| 45. 19.3 | 93. 11.9 |
| 46. .0000476 | 94. .377 |
| 47. 5.77 | 95. 1.56 |
| 48. 27.5 | 96. 9.24 |
| 49. 87.9 In. | 97. .604 |
| 50. 81.9 | 98. .560 |
| 52. 156. | 99. 38.2 |
| 53. .294 | 100. Square roots of numbers |
| 55. .00654 | from 110 to 130. |
| 56. .735 | Number Square Roots |
| 57. .615 | 110. 10.5 |
| 58. 13.6 | 111. 10.5 |
| 59. 77.9 | 112. 10.6 |
| 61. 19.6 | 113. 10.6 |
| 62. 21.4 | 114. 10.7 |
| 63. 33.1 | 115. 10.7 |
| 64. 13.9 | 116. 10.8 |
| 65. 10.5 | 117. 10.8 |
| 66. 56.7 | 118. 10.9 |
| 67. 1.6 mils. | 119. 10.9 |
| 72. 10.2 | 120. 11.0 |

- | | | | | |
|------|----------------------------|------|-----------------------------|--------------|
| 121. | 11.0 | 103. | 8.5 inches | (Use a 9-in. |
| 122. | 11 0 | | pipe, the nearest standard | size). |
| 123. | 11.1 | | Answers to test problems on | |
| 124. | 11.1 | | Page 16. | |
| 125. | 11.2 | | 1. | 3.15 |
| 126. | 11.2 | | 2. | 1.41 |
| 127. | 11.3 | | 3. | 11.4 |
| 128. | 11.3 | | 4. | 36.8 |
| 129. | 11.4 | | 5. | 13.5 |
| 130. | 11.4 | | | |
| 101. | 127 feet, 3 inches. | | | |
| 102. | 1.9 inches. | | | |
| | Use a 2-in. | | | |
| | pipe, the nearest standard | | | |
| | size. | | | |

ANSWERS.

Chapter II

Multiplication.

- | | | | |
|----|------|-----|-------------------|
| 1. | 7.39 | 6. | 273. |
| 2. | 19.3 | 7. | .0541 |
| 3. | 7.55 | 8. | .00167 |
| 4. | 58.5 | 9. | .0000910 |
| 5. | 258. | 10. | 12.56, 20.4 44.0. |

Cubes.

- | | | | |
|-----|------------------------------|-----|--------------|
| 11. | 2197. | 24. | 149000. |
| 12. | 2744. | 25. | 262000. |
| 13. | 3375. | 26. | 436,000,000. |
| 14. | 4096. | 27. | 12,500,000. |
| 15. | 4913. | 28. | 77,300,000. |
| 16. | 5832. | 29. | 679,000. |
| 17. | 6859. | 30. | 2,690,000. |
| 18. | 8000. | 31. | .04 |
| 19. | 9261. | 32. | .000185 |
| | (Three significant figures). | 33. | .000,000,314 |
| 20. | 10600. | 34. | 1.09 |
| 21. | 29800. | 35. | 9.53 |
| 22. | 97300. | 36. | 76.1 gal. |
| 23. | 104000. | | |

Cube Roots.

- | | | | |
|-----|-------|-----|-------|
| 1. | 1.44 | 12. | 1.94 |
| 2. | 3.107 | 13. | .832 |
| 3. | 6.69 | 14. | 6.22 |
| 4. | .669 | 15. | 15.66 |
| 5. | .3107 | 16. | 37.34 |
| 6. | .144 | 17. | .2535 |
| 7. | 13.77 | 18. | .211 |
| 8. | 3.628 | 19. | 1.012 |
| 9. | .922 | 20. | 47.7 |
| 10. | 35.59 | 21. | 20.4 |
| 11. | 3.68 | | |

Multiplication of More Than Two Numbers.

- | | | | |
|----|--------------|----|-------|
| 1. | 92.4 | 4. | 1.309 |
| 2. | 114.7 | 5. | 56.1 |
| 3. | 17,490,000 . | | |

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Combined Multiplication and Division.

1. .01815	8. .1585
2. .633	9. 4.58
3. 902.	10. 1.69
4. 328.	11. .0886
5. 1111.	12. .0786
6. 51.4	13. 1.073
7. .353	

Miscellaneous Calculations.

1. 32.3	7. 57,800,000.
2. 1.91	8. 1.234
3. .516	9. .408
4. .45	10. .00642
5. 1627.	11. 81.4
6. 35.8	12. 6.4

Sines and Cosines.

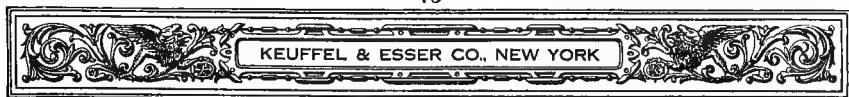
1. 1.	16. .250
2. .707	17. .585
3. .5	18. .937
4. .0523	19. .1435
5. .0116	20. .0276
6. .264	21. 19.
7. .0262	22. 15.1
8. .1478	23. 83.2
9. .0393	24. 32.0
10. .3665	25. 34.5
11. .1736	26. 16.3
12. .423	27. $a=9.11, h=8.04, c=6.49,$ $d=5.03.$
13. .743	28. $BC=4.70, BA=1.71$
14. .970	
15. .978	

Tangents.

1. .466	9. .270
2. .259	10. .1125
3. .713	11. .0306
4. .495	12. .911
5. .335	13. 4.82
6. 1.446	14. 29.0
7. 3.78	15. 31.9
8. .367	16. 30.4

Logarithms.

1. .093	6. 33.6
2. 1.736	7. 32,600,000,000.
3. 1.494, or 9.494-10.	8. 1.512
4. 2.8260, or 8.8260-10	9. 3.10
5. 2.866	10. 2.82



THE POLYPHASE SLIDE RULE.



No. 4053-3.

The Polyphase Slide Rule is especially devised for the solution of problems containing combinations of three factors and problems involving squares, square roots, cubes, cube roots, and many of the higher powers and roots. Furthermore, it offers in many cases a more ready and accurate means of performing the calculations which are possible with the ordinary Mannheim Slide Rule and it is especially adapted for electrical and hydraulic work.

Description. The Polyphase Slide Rule has in addition to the regular scales of the Mannheim, a scale of cubes (K) on the vertical edge of the rule, and an inverted scale (CI) on the face of the slide.

The cube or "K" scale consists of three consecutive and identical logarithmic scales, so placed that the extreme right and left indexes coincide with the right and left indexes of the "D" scale. Each of these three scales is graduated like the "C" and "D" scales and is one-third of their length.

The Inverted Scale (CI) consists of a complete logarithmic scale placed in reversed order, that is, prime 1 is at the right index, the scale progressing towards the left.

How to use the Manual with a Polyphase Rule.

Work all problems in Chapters 1, 2 and 3 according to directions for the use of the Mannheim Rule, with the exception of problems involving Reciprocals, Product of three factors, Cubes, Cube Roots, and Inverted Slide. Operations in these specific cases are performed more readily by the use of the special scales CI and K of the Polyphase Rule.

Use of Scales CI and K.

RECIPROCAL.

The reciprocal of any number on the CI scale will be found directly opposite on the C scale. For examples see page 33.

MULTIPLICATION.

Two Factors: Set one of the factors on the CI scale to the other factor on the D scale. Under the index on C find the product on D.

Example: $25 \times 36 = 900$.

Set 25 on CI to 36 on D. Under the right index of C find the product, 900, on D.

Three Factors: One of the many useful advantages of the Polyphase Rule is that it enables an operator to find the product of three factors with only one setting of the slide.

Example: $65 \times 24 \times 125 = 195,000$.

Set 65 on CI to 24 on D. Under 125 on C read 195,000 on D.

Compare with Example 2 on page 33, and note the advantage of the Polyphase over the Mannheim Rule.

Work the problems on page 34.

CUBES AND CUBE ROOTS.

Cubes. Since scale D is three times the length of each of the three scales which comprise the K scale, the values of the readings on the K scale are cubes of the coinciding values on the D scale, and conversely, the values on the D scale are cube roots of the coinciding values on the K scale. Therefore, to find the cube of a number, set the runner at the given number on D, when the coinciding number on K will be its cube.

Work the problems beginning on page 22, using the K scale and placing the decimal point by a rough calculation.

It will be found that the method of using the K scale for cubes and cube roots is more rapid, while the method given on page 22 involving scales A, B, C, and D is more accurate. Hence, even on the Polyphase Rule, when greater accuracy is required it may be worth while to take time to use scales A, B, C, and D.

Cube Roots. To find the cube root of a number:

When the number lies between 1 and 10, use the left hand scale of K.

When the number lies between 10 and 100, use the middle scale of K.

When the number lies between 100 and 1000, use the right hand scale of K.

When the number does not lie between 1 and 1000, follow the method of page 24.

Set the mark on the under side of the runner to the given number on K, selecting the proper one of the three scales as directed above. Opposite the runner line on D find the cube root.

Example 1. Find the cube root of 8.

Set the runner to 8 on the left hand scale of K. Opposite the runner line on D find 2, the cube root.

Example 2. Find the cube root of 90.

Set the runner to 90 on the middle scale of K. Opposite the runner line on D find 4.48, the cube root.

Example 3. Find the cube root of 900.

Set the runner to 900 on the right hand scale of K. Opposite the runner line on D find 9.66, the cube root.

Work the problem on page 24 beginning with Example 6, using scale K.

HIGHER POWERS AND ROOTS.

The fourth root of a number is obtained by finding the square root of the square root. The sixth root is obtained by finding the square root of the cube root. The eighth root is the square root of the fourth root.

Expressions Which May Be Read Directly By Means Of The Indicator, Without Setting The Slide:

1. $X = a^2$, set Runner to a on D, read X on A.
2. $X = a^3$, set Runner to a on D, read X on K.
3. $X = \sqrt{a}$, set Runner to a on A, read X on D.
4. $X = \sqrt[3]{a}$, set Runner to a on K, read X on D.
5. $X = \sqrt{a^2}$, set Runner to a on A, read X on K.
6. $X = \sqrt[3]{a^2}$, set Runner to a on K, read X on A.

RECIPROCAL.

7. $X = \frac{1}{a}$, set Runner to a on CI, read X on C.
8. $X = \frac{1}{a^2}$, set Runner to a on CI, read X on B.
9. $X = \frac{1}{\sqrt{a}}$, set Runner to a on B, read X on CI.
10. $X = \frac{1}{a^3}$, set Runner to a on CI, read X on K.
11. $X = \frac{1}{\sqrt[3]{a}}$, set Runner to a on K, read X on CI.

EXPRESSIONS SOLVED WITH ONE SETTING OF SLIDE.

ONE FACTOR.

12. $X = a^4$, set 1 to a on D, over a on C, read X on A.
13. $X = \frac{1}{a^4}$, set a on CI to a on D, under 1 on A, read X on B.
14. $X = a^5$, set a on CI to a on D, over a on B, read X on A.
15. $X = \frac{1}{a^6}$, set a on C to a on K, under a on CI, read X on K.
16. $X = a^6$, set 1 on C to a on D, under a on C, read X on K.
17. $X = a^7$, set a on CI to a on K, under a on C, read X on K.
18. $X = a^9$, set a on CI to a on D, under a on C, read X on K.
19. $X = \sqrt{a^5}$, set 1 to a on K, under a on B, read X on K.
20. $X = \sqrt{a^9}$, set 1 to a on A, under a on C, read X on K.
21. $X = \sqrt{a^{11}}$, set a on CI to a on K, under a on B, read X on K.
22. $X = \sqrt{a^{15}}$, set a on CI to a on D, under a on B, read X on K.
23. $X = \frac{1}{\sqrt[3]{a^2}}$, set 1 to a on K, under 1 on A, read X on B.
24. $X = \sqrt[3]{a^4}$, set 1 to a on K, under a on C, read X on D.
25. $X = \frac{1}{\sqrt[3]{a^4}}$, set a on CI to a on K, over 1 on D, read X on C.
26. $X = \sqrt[3]{a^5}$, set 1 to a on K, over a on B, read X on A.
27. $X = \frac{1}{\sqrt[3]{a^5}}$, set a on C to a on K, under a on CI, read X on D.
28. $X = \sqrt[3]{a^7}$, set a on CI to a on K, under a on C, read X on D.
29. $X = \sqrt[3]{a^8}$, set 1 to a on K, over a on C, read X on A.
30. $X = \frac{1}{\sqrt[3]{a^8}}$, set a on CI to a on K, under 1 on A, read X on B.
31. $X = \frac{1}{\sqrt[3]{a^{10}}}$, set a on C to a on K, over a on CI, read X on A.
32. $X = \sqrt[3]{a^{11}}$, set a on CI to a on K, over a on B, read X on A.
33. $X = \sqrt[3]{a^{14}}$, set a on CI to a on K, over a on C, read X on A.
34. $X = \sqrt[6]{a}$, set a on B to a on K, over 1 on D, read X on C.

35. $X = \frac{1}{\sqrt[6]{a}}$, set a on B to a on K, under 1 on C, read X on D.
36. $X = \sqrt[6]{a^5}$, set 1 to a on K, under a on B, read X on D.
37. $X = \sqrt[6]{a^7}$, set a on B to a on K, over a on D, read X on C.
38. $X = \frac{1}{\sqrt[6]{a^7}}$, set a on B to a on K, over a on D, read X on CI.
39. $X = \sqrt[6]{a^{11}}$, set a on CI to a on K, under a on B, read X on D.

SETTINGS FOR TWO FACTORS.

40. $X = ab$, set 1 to a on D, under b on C, read X on D.
41. $X = \frac{1}{ab}$, set a on CI to b on D, over 1 on D, read X on C.
42. $X = \frac{a}{b}$, set b on C to a on D, under 1 on C, read X on D.
43. $X = \frac{1}{\frac{a}{b}}$, set b on C to a on D, over 1 on D, read X on C.
44. $X = ab^3$, set 1 to a on A, over b on C, read X on A.
45. $X = \frac{1}{ab^3}$, set b on CI to a on A, under 1 on A, read X on B.
46. $X = \frac{a}{b^3}$, set b on C to a on A, over 1 on B, read X on A.
47. $X = \frac{a^3}{b}$, set a on C to b on A, under 1 on A, read X on B.
48. $X = a^3b^3$, set 1 on C to a on D, over b on C, read X on A.
49. $X = \frac{1}{a^3b^3}$, set a on CI to b on D, under 1 on A, read X on B.
50. $X = \frac{a^3}{b^3}$, set a on C to b on D, under 1 on A, read X on B.
51. $X = a\sqrt[3]{b}$, set 1 to a on D, under b on B, read X on D.
52. $X = \frac{1}{a\sqrt[3]{b}}$, set a on CI to b on A, over 1 on D, read X on C.
53. $X = \frac{\sqrt[3]{a}}{b}$, set b on C to a on A, under 1 on C, read X on D.
54. $X = \frac{a}{\sqrt[3]{b}}$, set b on B to a on D, under 1 on C, read X on D.
55. $X = a^3\sqrt[3]{b}$, set a on CI to a on D, under b on B, read X on D.
56. $X = \frac{a^3}{\sqrt[3]{b}}$, set b on B to a on D, under a on C, read X on D.
57. $X = \frac{\sqrt[3]{a}}{b^3}$, set b on C to a on A, under b on CI, read X on D.
58. $X = ab^3$, set 1 on C to a on K, under b on C, read X on K.
59. $X = \frac{a}{b^3}$, set b on C to a on K, under 1 on C, read X on K.
60. $X = \frac{a^3}{b^3}$, set a on CI to a on A, over b on CI, read X on A.

61. $X = \frac{a^2}{b^3}$, set b on B to a on D, over b on CI, read X on A.
62. $X = a^2 b^3$, set b on CI to a on D, over b on B, read X on A.
63. $X = a \sqrt[3]{b}$, set 1 on C to b on K, under a on C, read X on D.
64. $X = 1 \div a \sqrt[3]{b}$, set a on CI to b on K, over 1 on D, read X on C.
65. $X = a \div \sqrt[3]{b}$, set a on C to b on K, over 1 on D, read X on C.
66. $X = \sqrt[3]{a \div b}$, set b on C to a on K, under 1 on C, read X on D.
67. $X = a^3 b^3$, set 1 on C to b on D, under a on C, read X on K.
68. $X = \frac{a^3}{b^3}$, set b on C to a on D, under 1 on C read X on K.
69. $X = ab^4$, set b on CI to a on A, over b on C, read X on A.
70. $X = b^6 \sqrt{a^3}$, set b on CI to a on A, under b on C, read X on K.

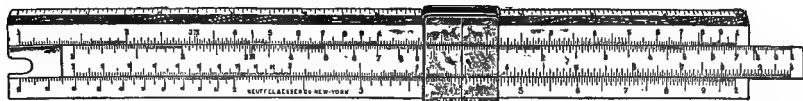
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SLIDE RULES OF ALL KINDS.

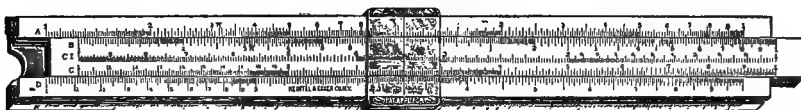
We are the largest manufacturers of slide rules in America and have the largest assortment. Some of the best known of our slide rules are shown below.

MANNHEIM



Mannheim Slide Rules can be used to advantage in every line of business. They have our patent adjustment for obtaining any desired friction of the slide.

POLYPHASE

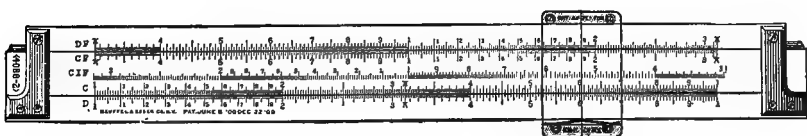


The Polyphase Slide Rule is of the Mannheim type, but has an inverted C scale and a scale of cubes in addition to the regular Mannheim scales. This arrangement facilitates the solution of many problems involving three factors, as well as many powers and roots.

DUPLEX

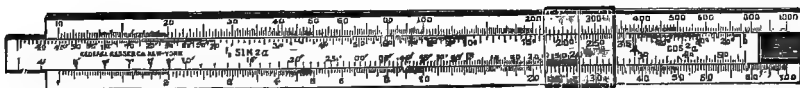
Duplex Slide Rules are practically two rules in one, with all the scales on the exterior faces, where they may be readily seen and utilized by merely turning the rule over.

POLYPHASE DUPLEX



The Polyphase Duplex Slide Rules are a combination of the Polyphase and the Duplex Rules, with the addition of several special scales.

K & E STADIA SLIDE RULE

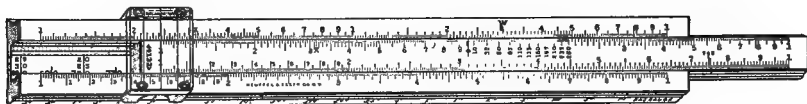


This form of Stadia Slide Rule is remarkable for its simplicity. By one setting of the slide, the horizontal distance and vertical height can be obtained at once when the stadia rod reading and the angle of elevation or depression of the telescope are known.

SLIDE RULES FOR MANY PURPOSES.

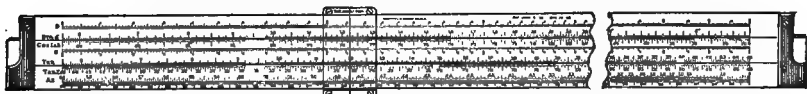
In addition to our extended line of regular Mannheim and Duplex Slide Rules, we manufacture a variety of rules for special purposes, of which we name a few below:

THE ROYLANCE ELECTRICAL SLIDE RULE



This rule is a modification of our regular Slide Rule No. 4035 and can be used for all the calculations made with the ordinary Slide Rule. In addition to the usual scales, it carries a series of scales or gauge marks by means of which the different properties of copper wire, such as size, conductivity, weight, etc., may be determined without the use of tables.

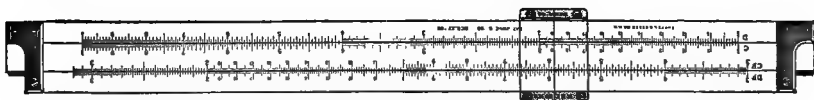
SURVEYOR'S DUPLEX SLIDE RULE



The fact that all astronomical data essential to surveying, such as azimuth, time, latitude, etc., can be ascertained by means of the usual type of Transit with vertical circle but without solar attachment, while generally known, is rather seldom utilized in this country. The main reason for this surprising condition is the difficulty of computing, in the field, by spherical trigonometry, the results of observations.

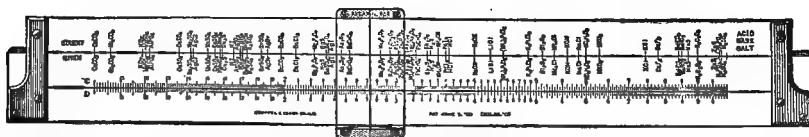
The new K & E Surveyor's Slide Rule entirely eliminates this difficulty by reducing the hitherto complicated calculations to mere mechanical operations, thereby rendering the method of field astronomy with the regular Engineer's Transit extremely simple and practical.

MERCHANT'S (CALCULATING) SLIDE RULE



Especially designed for the merchant, importer, exporter, accountant, manager, mechanic, foreman, etc. By means of it, all manner of problems involving multiplication, division and proportion can be correctly solved without mental strain and in a small fraction of the time required to work them out by the usual "figuring".

THE CHEMIST'S DUPLEX SLIDE RULE



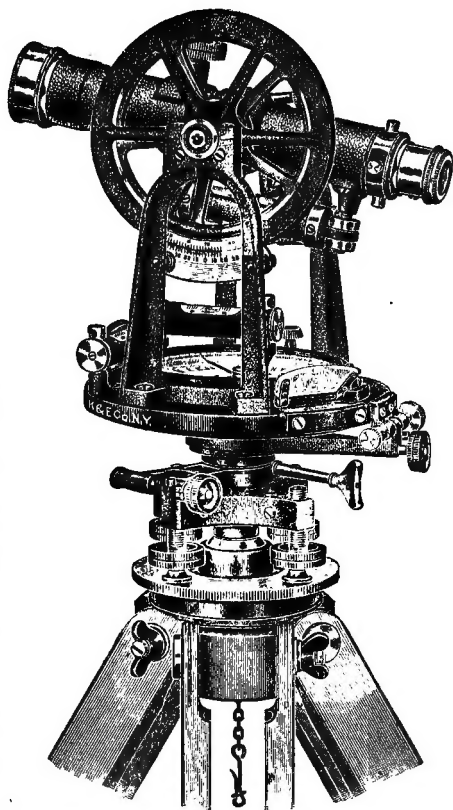
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